1. Let $f(x) = ax^5 - bx^3 + cx - 10$ If $f(7) = 12$, find $f(-7)$.

   A. -32

   B. -12

   C. 10

   D. 12

   E. None of the above

2. If $x^y = 1/2$, then $(x^2)^{3y} = ?$

   A. $\frac{1}{8}$

   B. $\frac{1}{32}$

   C. $\frac{1}{64}$

   D. 8

   E. 32

3. If $\log_b 2 = x$, $\log_b 3 = y$, and $\log_b 5 = z$, then $\log_b(0.075) = ?$

   A. $y - 3x + z$

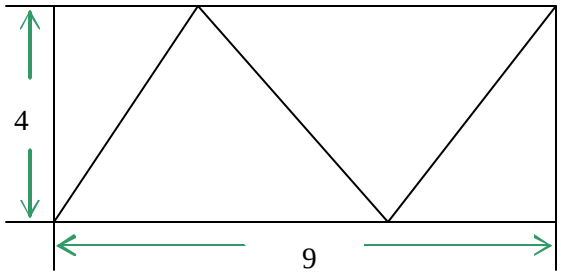
   B. $y - 3x - z$

   C. $y - x^3 + z$

   D. $y - x^3 - z$

   E. None of the above.

4. If the three segments inside the rectangle have the same length, then the sum of their lengths is:



A. 13

B. 12

C. 18

D. 15

E. 10

5. Find $x^2 + y^2$ if x and y are positive integers such that $xy + x + y = 71$ and $x^2 y + xy^2 = 880$

A. 146

B. 130

C. 61

D. 208

E. 100

6. **(Tie Break No.1)** If $\tan^{-1} 2x + \tan^{-1} 3x = \frac{p}{4}$, what is the value of x ?

A. $\{-1\}$

B. $\left\{\frac{1}{6}, -1\right\}$

C. $\left\{\frac{p}{10}, \frac{3p}{20}\right\}$

D. $\left\{-\frac{1}{6}, 1\right\}$

E. $\left\{\frac{1}{6}\right\}$

7. If $f(2x+1) = 4x^2 + 2x - 6$, find the sum of the zeros of $f(x)$.

   A. 4

   B. -2

   C. 1

   D. 3

   E. 5

8. The sum of the first three terms of a geometric sequence of positive integers is equal to seven times the first term. The sum of the first four terms is 45. What is the first term of the sequence?

   A. 1

   B. 2

   C. 3

   D. 4

   E. 5

9. Three solid balls, each of radius 3.25 *cm*, are stored in a circular cylindrical can with the smallest possible radius and volume. What fraction of the can's volume is air?

   A. $1/4$

   B. $2/3$

   C. $4/5$

   D. $3/5$

   E. $1/3$

10. Let p be an odd whole number and let n be any whole number. Which of the following statements about the whole number $\left(p^2 + np\right)$ is always true?

A. It is always odd

B. It is always even

C. It is even only if n is even

D. It is odd only if n is odd

E. It is odd only if n is even

11. If $i^2 = -1$, then $(1-i)^{11} = ?$

A. $32 + 32i$

B $-32 - 32i$

C. $32 - 32i$

D. $-32 + 32i$

E. None of the above

12. Given $f(x) = \log \dfrac{1-x}{1+x}$, if $f(a) = b$, then $f(-a) = ?$

A. b

B. $-b$

C. $\dfrac{1}{b}$

D. $-\dfrac{1}{b}$

E. None of the above

13. Which of the following is the solution set of the inequality $1 < |2x+1| < 3$?

A. $(-1, 0) \cup (1, 2)$

B. $(0, 1)$

C. $(-2, 0)$

D. $(-2, -1) \cup (0, 1)$

E. $(-2, -1)$

14. If $\sin 2q < 0$ and $\cos q - \sin q < 0$, then q is terminated in which of the following quadrants?

A. I

B. II

C. III

D. IV

E. It can not be determined

15. **(Tie Break No. 2)** Given $a^2 + b^2 = 1$, $b^2 + c^2 = 2$, and $c^2 + a^2 = 2$, find the minimum value of $ab + bc + ca$

A. $\sqrt{3} - 1/2$

B. $-1/2 - \sqrt{3}$

C. $-5/2$

D. $1/2 + \sqrt{3}$

E. $1/2 - \sqrt{3}$

16. Find the maximum value of the function $f(x) = \dfrac{1}{1-x(1-x)}$.

A. $4/3$

B. $3/4$

C. $5/4$

D. $4/5$

E. 1

17. A sphere is inscribed in a right circular cone. If the height of the cone is 3 times the radius of the sphere, what is the ratio of the volume of the sphere to the volume of the cone?

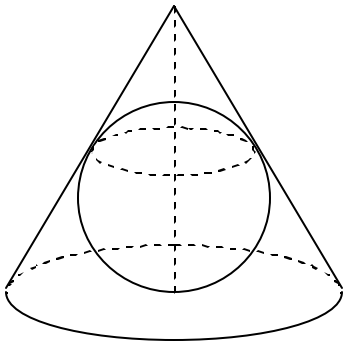
A. $4:9$

B. $2:3$

C. $4:3\sqrt{3}$

D. $2\sqrt{3}:9$

E. $1:\sqrt{3}$



18. The function $f(x) = x^2 - 2ax - 3$ ($x \in [1, 2]$) has an inverse function, if and only if a satisfies which of the following?

A. $a \in (-\infty, 1]$

B. $a \in [2, \infty)$

C. $a \in [1, 2]$

D. $a \in (-\infty, 1] \cup [2, \infty)$

E. $a \in (-\infty, -2] \cup [-1, \infty)$

19. If the real numbers *a*, *b*, and *c* satisfy $c < b < a$ and $ac < 0$, Which of following statement is not necessarily true?

A. $ab > ac$

B. $c(b-a) > 0$

C. $cb^2 < ab^2$

D. $ac(a-c) < 0$

E. None of the above

20. There are two ways to inscribe a square into an isosceles right triangle. (See the figure (1) and (2).) Find the ratio of the area of the square in the figure (1) to the area of the square in the figure (2) if these two isosceles right triangles are congruent.

A. $4:3$

B. $9:8$

C. $8:7$

D. $7:6$

E. $3:2$

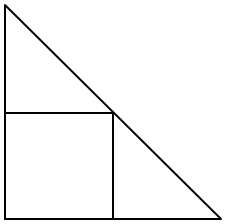


Figure (1)

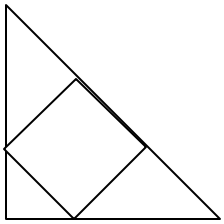


Figure (2)

21. If a is a real number and in the expansion of $(x+a)^{10}$, the coefficient of x^7 is -15, then $a = ?$

A. $-1/4$

B. -2

C. $1/4$

D. $-1/2$

E. $1/2$

22. **(Tie Break No. 3)** Circles P and L are tangent and have radii 9 and 4 respectively. Find the length of the common tangent GN.

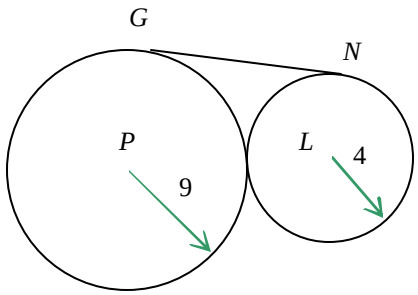
A. 9

B. 10

C. 11

D. 12

E. 13



23. If two marbles are removed at random from a bag containing only black and white marbles, the chance that they are both white is 1/3. If three are removed at random, the chance that they are all white is 1/6. What is the ratio of black balls to white balls?

A. $1:3$

B. $3:4$

C. $1:4$

D. $2:3$

E. $1:2$

24. Find $i+2i^2+3i^3+\ldots+2004i^{2004}+2005i^{2005}$

A. $1002-1002i$

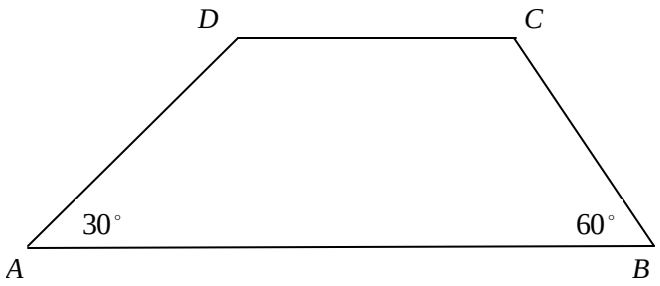
B. $1002+1003i$

C. $-1004+1003i$

D. $-1004-1004i$

E. None of the above

25. Express the perimeter of trapezoid *ABCD* in simplest exact form if $DC = 4$ and $AD = 6$.



A. $20 + 6\sqrt{3}$

B. $20 + 4\sqrt{3}$

C. $14 + 4\sqrt{3}$

D. Not enough information

E. None of the above

26. What is the solution to the following equation?
$4 = 1 + 2x + 4x^2 + 8x^3 + 16x^4 + ... + 2^n x^n + ...$

A. No solution exists

B. $1/4$

C. $3/8$

D. $-3/4$

E. $3/4$

27. A ball is dropped from a height of 24 feet. Each time it drops *h* feet, it rebounds $\frac{2}{3}h$ feet. Find the total vertical distance traveled by the ball.

A. 72 feet

B. 84 feet

C. 120 feet

D. 144 feet

E. None of the above

28. If $n \geq 2$ is a natural number, which of the following integers **must** be divisible by 3?

A. $n(n^2-1)$

B. $n^2(n+1)$

C. n^2+3n+2

D. All of the above

E. None of the above

29. **(Tie Break No. 4)** Let AB be a diameter of a circle with length 26. Let C and D be located on AB such that $AC = 1$ and $AD = 8$. Let E and F be points on one of the arcs AB for which EC and FD are perpendicular to AB. Find EF.

A. 5

B $5\sqrt{2}$.

C. 7

D. $7\sqrt{2}$

E. 12

30. Find the range of the function $f(x) = \dfrac{6\cos^4 x + 5\sin^2 x - 4}{\cos 2x}$

A. $\{y \mid -1 \leq y \leq 2\}$

B. $\{y \mid -2 \leq y \leq 1\}$

C. $\left\{y \mid -1 \leq y < \dfrac{1}{2} \text{ or } \dfrac{1}{2} < y \leq 2\right\}$

D. $\left\{y \mid -2 \leq y < \dfrac{1}{2} \text{ or } \dfrac{1}{2} < y \leq 1\right\}$

E. None of the above

31. Express $\sin 3x$ in terms of $\sin x$.

A. $3\sin x + 4\sin^3 x$

B. $3\sin x - 4\sin^3 x$

C. $-3\sin x + 4\sin^3 x$

D. $-3\sin x - 4\sin^3 x$

E. None of the above.

32. If a tangent line is drawn from the point $(2,-3)$ to the circle $x^2 + y^2 - 2x - 4y - 1 = 0$, find the distance from $(2,-3)$ to the point of tangency.

A. $\sqrt{26}$

B. $4\sqrt{2}$

C. $2\sqrt{5}$

D. 4

E. 6

33. Find the exact value of $\tan(\cos^{-1}(\sin 30^\circ))$

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{3}$

C. $\sqrt{3}$

D. $\frac{\sqrt{3}}{2}$

E. None of the above

34. Let x be a real number such that $\sec x - \tan x = 2$. Find $\sec x + \tan x$.

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. 2

E. None of the above

35. The graph of $f(x) = \cot(4x + f)$ passes through the point $(p/6, 0)$. Which of the following could be a value of f?

A. $p/4$

B. $-p/4$

C. $p/3$

D. $-p/3$

E. $-p/6$

36. Find the minimum value of $y = \sin x + \cos x$.

A. -1

B. $-\sqrt{2}$

C. -2

D. 0

E. 1

37. **(Tie Break No. 5)** Let $f_n(x) = \sin^n x + \cos^n x$. For how many values of x in $[0, p]$ is it true that $6 f_4(x) - 4 f_6(x) = 2 f_2(x)$?

A. 2

B. 4

C. 6

D. 8

E. More than 8

38. Find the following sum: $\sum_{n=1}^{\infty} \frac{1}{n^2 + 4n + 3}$

A. 0

B. $\frac{5}{6}$

C. $\frac{5}{8}$

D. $\frac{5}{12}$

E. Sum does not exist

39. In right triangle ABC, if $AD = DB + 8$, what is the value of x?

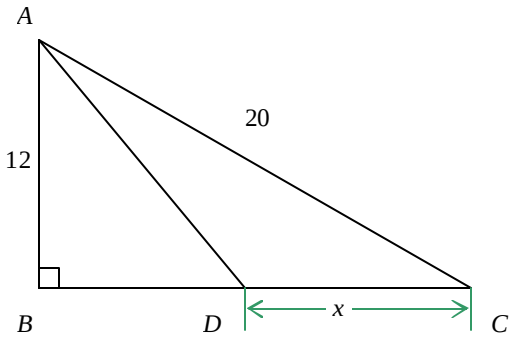
A. 16

B. 11

C. 5

D. 8

E. None of the above



40. Solve the following equation for x: $\sin^{-1} x - \cos^{-1} x = \frac{p}{6}$

A. $\{1/2\}$

B. $\{-1/2\}$

C. $\{\sqrt{3}/2\}$

D. $\{-\sqrt{3}/2\}$

E. $\{\pm\sqrt{3}/2\}$

41. A multiple-choice examination consists of 20 questions. The scoring is $+5$ for each correct answer, -2 for each incorrect answer, and 0 for each unanswered question. John's score is 48. What is the maximum number of questions he could have answered correctly?

A. 9

B. 10

C. 11

D. 12

E. 16

42. Find $\lim_{x\to 0} \frac{\cos 2x - 1}{x^2}$

A. -2

B. 2

C. 0

D. $-1/2$

E. Does not exist

43. A speaker talked for sixty minutes to a full auditorium. Twenty percent of the audience heard the entire talk. Ten percent slept through the entire talk. Half of the remainder heard one-third of the talk and the other half heard two-thirds of the talk. What is the average number of minutes of the talk heard by members of the audience?

A. 24

B. 27

C. 30

D. 33

E. 36

44. An international mathematics conference was held at a neutral site. A total of 15 delegates were from Japan, England, United States, and Russia. Each country sent a different number of delegates, and each was represented by at least one delegate. The United States and England sent a total of 6 delegates. One country sent exactly four delegates. England and Russia sent a total of 7 delegates. Which country sent the most number of delegates?

A. Russia

B. Japan

C. United States

D. England

E. Insufficient information given.

45. The areas of two similar triangles are 45 cm^2 and 80 cm^2. The sum of their perimeters is 35 cm. Find the perimeters of the triangles.

A. 15 cm and 20 cm

B. 14.5 cm and 20.5 cm

C. 18 cm and 17 cm

D. 30 cm and 5 cm

E. 25 cm and 10 cm