Math Bowl 2006 Written Test

1. If $xy = -3$ and $x^2 + y^2 = 12$, then $(x-y)^2 = ?$

   A. 25

   B. 18

   C. 16

   D. 4

   E. 11

2. In ΔABC, $m\angle C = 120°$, $a = 3$, and $b = 5$. Find $m\angle A$

   A. $\sin^{-1}\left(\frac{3\sqrt{57}}{38}\right)$

   B. $\sin^{-1}\left(\frac{5\sqrt{57}}{38}\right)$

   C. $\sin^{-1}\left(\frac{3\sqrt{3}}{14}\right)$

   D. $\sin^{-1}\left(\frac{5\sqrt{3}}{14}\right)$

   E. None of the above

3. A regular hexagon and an equilateral triangle have the same perimeter. What is the ratio of the area of the hexagon to the area of the triangle?

   A. 2:3

   B. 4:1

   C. 3:2

   D. 1:4

   E. 5:3

4. The period of a simple pendulum is directly proportional to the square root of its length. If a pendulum has a length of 6 feet and a period of 2 seconds, to what length should it be shortened to achieve a 1 second period?

A. 2.5 *ft*.

B. 1.5 *ft*

C. 2.0 *ft*.

D. 3.5 *ft*.

E. 0.5 *ft*.

5. If $x^2 - x - 1$ divides $ax^6 + bx^5 + 1$ evenly, find the sum of a and b.

A. 3

B. -2

C. 5

D. -5

E. 0

6. Given a circle of radius r with a rectangle $ABCO$ inscribed in one quadrant, what is the length of diagonal AC?

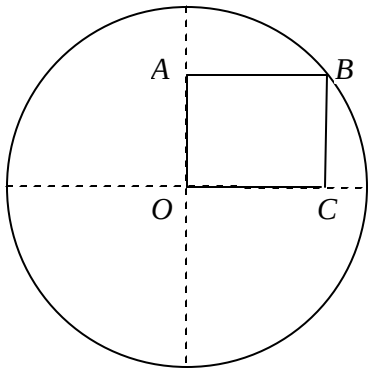
A. $\frac{3p}{4}r^2$

B. $\frac{2p}{3}r$

C. $\frac{p}{4}r^2$

D. $r + \frac{p}{8}$

E. r



7. The number of horses and sheep in a certain town is such that the ratio of the difference to the sum is 1:7 and the ratio of the sum to the product is 7:24. What is the sum of the number of horses and sheep?

   A. 20

   B. 14

   C. 10

   D. 17

   E. 24

8. Tom can plow a field in 12 hours, but with Pat helping him they can plow the field together in 8 hours. If Pat works alone plowing for 12 hours, how long will it take Tom working alone to plow the remainder of the field?

   A. 3 hours

   B. 4 hours

   C. $4\frac{1}{2}$ hours

   D. 6 hours

   E. $6\frac{2}{3}$ hours

9. The will of an eccentric millionaire reads as follows: "I leave $4/17$ of my estate to my son, $7/13$ of the remainder to my wife, $2/3$ of this remainder to my daughter, and the remaining \$2,000,000 to my dog." What was the total amount of the estate?

   A. \$10,000,000

   B. \$20,000,000

   C. \$15,000,000

   D. \$18,000,000

   E. \$17,000,000

10. A survey of 40 shoppers found that 12 liked Brand X, 23 liked Brand Y, and twice as many disliked both brands as liked both brands. How many liked both brands?

A. 3

B. 8

C. 2

D. 5

E. 10

11. **(Tie Break No.1)** The radius of the quarter-circle is 2. The two semi-circles are tangent to each other. Find the radius of the smaller semicircle.

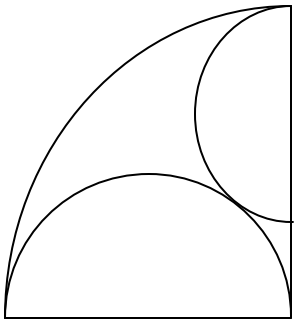
A. $1/2$

B. $2/3$

C. $3/4$

D. $1/4$

E. 1



12. In a right triangle ΔABC, $C = 90^\circ$. Which of the following statements is NOT true?

A. $0 < \sin A + \sin B \le \sqrt{2}$

B. $\sin^2 A + \sin^2 B = \sin^2 C$

C. $\cos^2 A + \cos^2 B = 1$

D. $\tan \dfrac{A+B}{2} = \sin C$

E. At least one of the above statements is not true.

13. **(Tie Break No.2)** What is the value of the expression

$$\frac{1}{\log_2 100!}+\frac{1}{\log_3 100!}+\frac{1}{\log_4 100!}+\cdots+\frac{1}{\log_{100} 100!}?$$

A. 1

B. 0.1

C. 0.01

D. 10

E. None of the above

14. If $x^2 = x+5$, then $x^3 = ?$

A. $6x+5$

B. $x+10$

C. $6x^2+5$

D. x^2+5x+5

E. None of the above

15. If the square of the geometric mean of x and y is equal to the sum of x and y, express y in terms of x.

A. $y=\frac{1}{x-1}$

B. $y=\frac{x}{x-1}$

C. $y=\frac{1}{x}+1$

D. $y=\frac{1}{x^2-1}$

E. $y=\frac{x}{x+1}$

16. **(Tie Break No.3)** The library in Destin has between 1000 and 2000 books. Of these, exactly 25% are fiction, $\frac{1}{13}$ are biographies, and $\frac{1}{17}$ are atlases. How many books are either biographies or atlases?

A. 240

B. 250

C. 270

D. 280

E. None of the above.

17. If n is an integer, which of the following must be true?

A. $3n+1$ is odd

B. $n(n+2)$ is even

C. $n(3n+3)$ is divisible by 6

D. $n(n+1)$ is divisible by 3

E. $n(n+3)$ is divisible by 4

18. $x(x(x(x+2)+2)+2)+2=?$

A. $x^4+2x^3+2x^2+2x+2$

B. $x^4+2x^3+4x^2+2x+2$

C. $x^4+2x^3+8x^2+2x+2$

D. $x^4+4x^3+16x^2+8x+4$

E. None of the above

19. Let $ABCD$ be a square piece of paper. A is folded onto C then B is folded onto D. The area of the resulting figure is 9 square inches. Find the perimeter of the square $ABCD$.

A. 9 inches

B. 16 inches

C. 18 inches

D. 24 inches

E. 36 inches

20. If $f(x) = \sqrt[3]{x}$ and $g(x) = x - 5$, find $\left(f^{-1} \circ g^{-1}\right)(-3)$

A. -6

B. -4

C. 2

D. -8

E. None of the above

21. If a 1-inch thick slab is sliced off one end of a cube, 294 cubic inches remain. What is the length of an edge of this cube?

A. $3+\sqrt{5}$

B. $5\sqrt{3}$

C. 7

D. $11\sqrt{2}$

E. 12

22. Let $A=\begin{bmatrix}1 & 1 & z\\ 0 & 1 & 0\\ x & -1 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & -4 & -1\\ 0 & y & 0\\ -2 & 3 & 1\end{bmatrix}$, find the values of x, y, and z so that $B=A^{-1}$. Then $x+y+z=?$

A. 0

B. 1

C. -1

D. 4

E. None of the above

23. In the figure, it is given that angle C = 90°, $\overline{AD}=\overline{DB}$, DE perpendicular to AB, $\overline{AB}=20$, and $\overline{AC}=12$. Find the area of the quadrilateral ADEC.

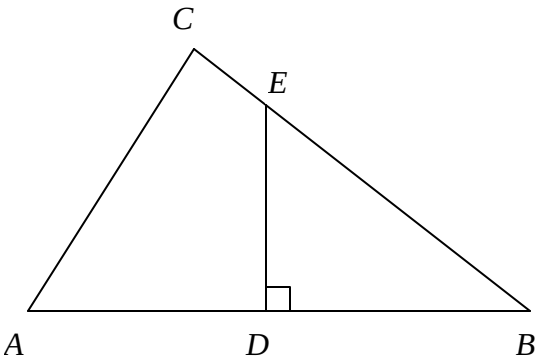
A. 75

B. 58.5

C. 48

D. 37.5

E. None of the above



24. Find the exact value of $\tan\left(\cos^{-1}\frac{1}{3}-\sin^{-1}\frac{1}{3}\right)$

A. 0

B. $\frac{7}{4\sqrt{2}}$

C. $4\sqrt{2}$

D. $\frac{4\sqrt{2}}{7}$

E. 1

25. The sum of the solutions of $(\log_4 x)^2 = \log_4(x^2)$ is equal to?

A. 2

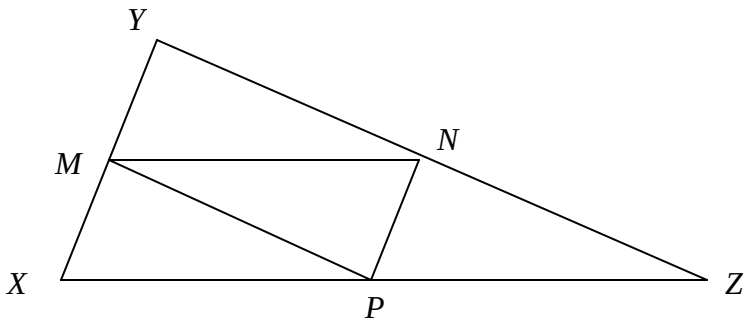
B. 3

C. 12

D. 16

E. 17

26. In ΔXYZ, points *M*, *N*, and *P* are midpoints. If $XY = 10$, $YZ = 15$, and $XZ = 17$, what is the perimeter of ΔMNP?



A. $10\frac{2}{3}$

B. 14

C. 16

D. 21

E. Cannot be determined

27. If $f(x^5) = \log_a x$, then $f(2) = ?$

A. $\log_a 2$

B. $\log_a 32$

C. $\log_a \frac{1}{32}$

D. $\frac{\log_a 2}{5}$

E. $5\log_a \frac{1}{2}$

28. Find $3a + 2b + c$ if the parabola $y = ax^2 + bx + c$ passes through the points $(1, -2)$, $(-2, 19)$, and $(3, 4)$.

A. 0

B. 1

C. -2

D. 3

E. -3

29. Simplify $\left(\dfrac{(x+1)^2 (x^2 - x + 1)^2}{x^3 + 1} \right)^2 \cdot \left(\dfrac{(x-1)^2 (x^2 + x + 1)^2}{x^3 - 1} \right)^2$

A. 1

B. $(x+1)^4$

C. $(x^3 + 1)^4$

D. $(x^6 - 1)^2$

E. $(x^6 + 1)^4$

30. In the figure, if the radius OA of the circle is 6 and AB is tangent to the circle what is the area of the shaded part?

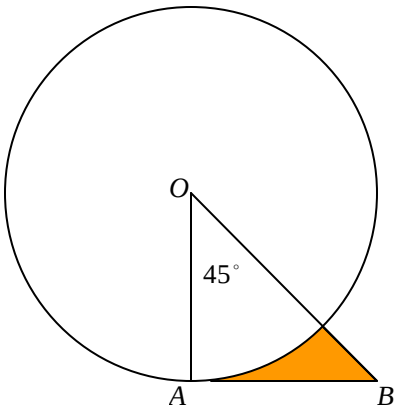
A. $36 - \dfrac{3p}{2}$

B. $18 - \dfrac{9p}{2}$

C. $18 - \dfrac{3p}{2}$

D. $18 - 3p$

E. $36 - 9p$



31. It is given that $\left(r+\frac{1}{r} \right)^2 = 3$. Find the value of $r^3 + \frac{1}{r^3}$

A. 1

B. 2

C. 0

D. 3

E. 6

32. A rectangular box with a square base has the property that the ratio of its volume to its surface area is 1. If one side of the square base has length a units, express the height b in terms of a.

A. $\frac{2a}{a-4}$

B. $\frac{4a}{a-2}$

C. $\frac{2a}{a+4}$

D. $\frac{4a}{a-2}$

E. None of the above.

33. Let $F(x) = \sqrt{\sin^4 x + 4\cos^2 x} - \sqrt{\cos^4 x + 4\sin^2 x}$. Which of the following expressions is equivalent to $F(x)$?

A. $1 - \sqrt{2}\sin x$

B. $-1 + \sqrt{2}\cos x$

C. $\cos x - \sin x$

D. $\cos 2x$

E. None of the above

34. Find the exact value of $\tan\left(\frac{5p}{12}\right)$.

A. 3

B. $\sqrt{3}$

C. $1+\sqrt{2}$

D. $\frac{\sqrt{3}+1}{2}$

E. $2+\sqrt{3}$

35. **(Tie Break No.4)** In the figure, two circles have the same radius 6 and the center of one circle is on the other one. Find the area of the shaded region.

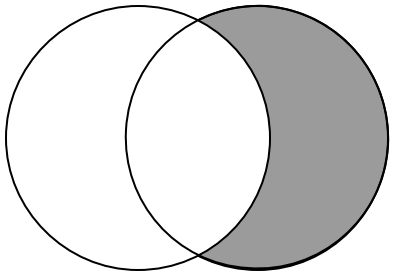
A. $12p-6\sqrt{3}$

B. $24p-18\sqrt{3}$

C. $12p+18\sqrt{3}$

D. $36p-18\sqrt{3}$

E. $12p+27\sqrt{3}$



36. How many ordered triples of positive integers (x,y,z) satisfy $\left(x^y\right)^z=64$

A. 6

B. 7

C. 8

D. 9

E. None of the above

37. Given $r = \dfrac{3}{4\cos q - \sin q}$, find an equivalent equation in rectangular coordinates.

A. $y = \dfrac{3}{4 - x}$

B. $4x - y = 3$

C. $3y + 4 = -1$

D. $3y + 4 = -x$

E. It is not possible to write the equation in rectangular coordinates.

38. **(Tie Break No.5)** Find the minimum value of the function $f(x) = \dfrac{1 - \cos 2x + 4\cos^2 x}{\sin 2x}$ on the open interval $(0, p/2)$

A. 3

B. $2\sqrt{2}$

C. 2

D. $2\sqrt{3}$

E. 4

.

39. $\sin^{-1}(\sin(p + 1)) =$

A. 1

B. -1

C. $p + 1$

D. $p - 1$

E. None of the above.

40. If $x = 2\sin q$ and $0 \le q < \frac{p}{2}$, then $\ln|\sec q + \tan q| - \ln|\cos q| = ?$

A. $\ln\left|\frac{x}{\sqrt{4-x^2}}\right|$

B. $\ln\left|\frac{4}{\sqrt{4-x^2}}\right|$

C. $\ln\left|\frac{2x}{\sqrt{4-x^2}}\right|$

D. $\ln\left|\frac{2}{2-x}\right|$

E. None of the above

41. Let $F(x) = g(3x^2)$ and $g(x)$ be a differentiable function. If $g'(12) = 5$ and $g(12) = -3$, find $F'(2)$.

A. 5

B. -3

C. 60

D. -36

E. None of the above

42. If $\cos x = \frac{3}{5}$ and $\cot x$ is negative, find the value of $\frac{\sin x - \tan x}{1 + \sec x}$.

A. 0

B. $1/5$

C. $-4/5$

D. $4/5$

E. $-1/5$

43. The oracle of Niceville tells the truth whenever she chooses to answer a question, and the probability that she will choose to answer a question is 1/3. A student plans to ask the oracle 3 times for his grade on the semester exam. What is the probability that he will get the answer?

A. 2/3

B. 19/27

C. 5/9

D. 1

E. 7/8

44. The perimeter of a rectangle is 100 and its diagonal has length x. What is the area of the rectangle?

A. $625 - x^2$

B. $625 - \frac{x^2}{2}$

C. $1250 - x^2$

D. $1250 - \frac{x^2}{2}$

E. None of the above

45. The area of the circle formed by the intersection of a sphere and a plane is p. If the distance from the center of the sphere to the plane is 1, find the surface area of the sphere.

A. $8\sqrt{2}p$

B. $4\sqrt{2}p$

C. $8p$

D. $4p$

E. $16p$