1. Solve the inequality $\log_{\frac{1}{5}} \frac{4x+6}{x} \geq 0$.

   A. $(1, \infty)$

   B. $[-2, 0)$

   C. $\left(-\infty, -\frac{3}{2}\right) \cup (0, \infty)$

   D. $\left[-2, -\frac{3}{2}\right)$

   E. $\left(-\frac{3}{2}, 0\right)$

2. What is the perimeter of the figure shown, given that there is a right angle at each corner and that two of the sides have lengths 12 and 16 as indicated?

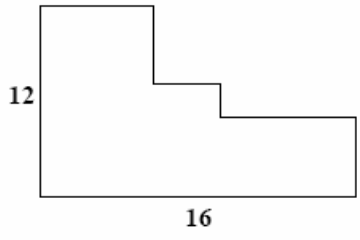
   A. 50

   B. 52

   C. 54

   D. 56

   E. 58



3. Find the area of the shaded region in the figure if the center of the circle is *O* and the radius of the circle is 6.

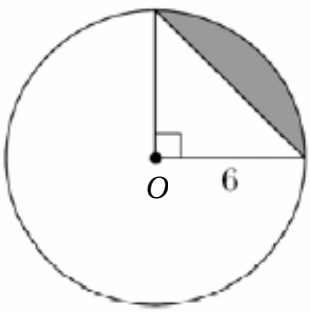
   A. $9p - 18$

   B. $\frac{9p}{2} + 6$

   C. $\frac{3p + 9}{2}$

   D. $\frac{6p - 3}{2}$

   E. $\frac{p - 6}{3}$



4. Solve the inequality: $|x^2 - 5x| < 6$

   A. $(-1,2) \cup (3,6)$

   B. $(0,2) \cup (3,6)$

   C. $(-1,6)$

   D. $(0,\infty)$

   E. $(-2,3)$

5. A convict escapes from a prison and has a half-hour's start on two guards and a bloodhound that race after him on his track. The guards' speed is four miles per hour. The dog's speed is 12 miles per hour. The prisoner can do only 3 miles per hour. The dog runs up to the prisoner and then back to the guards, and so on back and forth until the guards catch the prisoner. How far does the dog travel altogether?

   A. 24 miles

   B. 6 miles

   C. 12 miles

   D. 18 miles

   E. None of the above.

6. Two points are picked at random on the circle $x^2 + y^2 = 1$. What is the probability that the chord joining the two points has a length of at least 1?

   A. $1/4$

   B. $1/3$

   C. $1/2$

   D. $2/3$

   E. $3/4$

7. It is given that $\left(r+\frac{1}{r}\right)^2=3$. Find the value of $r^3+\frac{1}{r^3}$.

A. 1

B. 2

C. 0

D. 3

E. 6

8. A square is inscribed in another square, such that each vertex divides a side of the outside square into intervals of length x and y, where $x>y$. What is x/y if the area of the inscribed square is 2/3 of the area of the outside square?

A. 3

B. $2+\sqrt{2}$

C. $2+\sqrt{3}$

D. 4

E. $2+\sqrt{5}$

9. If $f(x)=\frac{x(x-1)}{2}$, then $f(x+2)$ equals:

A. $x+2f(x)$

B. $f(x)+f(2)$

C. $x^2+2f(x)$

D. $\frac{xf(x)}{x+2}$

E. $\frac{(x+2)f(x+1)}{x}$

10. Given three rectangles of area *A*, with the following dimensions, find $x - y$.

	Rectangle (i)	Rectangle (ii)	Rectangle (iii)
Length:	x	$x-3$	$x+3$
Width:	y	$y+2$	$y-1$

A. 9

B. 5

C. 4

D. 36

E. 1

11.(**Tie Break No.1**) Define the operations $a*b$ and $n\#$ as follows:

$$a*b = a + \frac{b^2 - ab}{a-b} \text{ and } n\# = n*([n-1]*([n-2]*(...*(3*(2*1)))))$$

Given that $\begin{bmatrix} 6\# & 7\# \\ 13\# & -15\# \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, find $x + y$.

A. 1

B. $\frac{3}{4}$

C. $-\frac{49}{4}$

D. $\frac{7}{4}$

E. None of the above.

12. If *x* is a real number and $4^x + 4^{-x} = 7$, then find the value of $8^x + 8^{-x}$.

A. 18

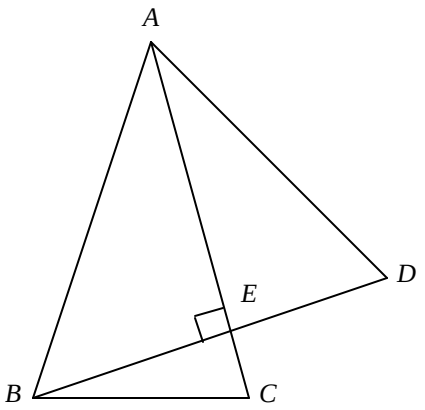
B. 14

C. 49

D. 54

E. Not enough information is provided

13. (**Tie Break No.2**) Triangles ΔABC and ΔABD are isosceles with $AC = AB = BD$, and the side BD intersects the side AC at E. If BD is perpendicular to AC, then $\angle C + \angle D$ is



A. 115°

B. 120°

C. 130°

D. 135°

E. Not uniquely determined

14. If $x^2 - x = 5$, then $x^4 - x = ?$

A. $25 - 2x$

B. $10(3 + 10x)$

C. $3(1 + x^2)$

D. $10(3 + x)$

E. $5(5 + x)$

15. Which of the following polynomials $p(x)$ has the property that $\sqrt{5} - \sqrt{3}$ is a solution to the equation $p(x) = 0$?

A. $p(x) = 3x^2 + 15x + 5$

B. $p(x) = x^4 - 16x^2 + 4$

C. $p(x) = x^4 + 3x^2 - 5$

D. $p(x) = x^3 - 3x^2 + 15$

E. $p(x) = x^4 - 8x^2 + 6$

16. In the figure on the right side, $AD = 3$, $DB = 5$, $BC = 7$ and DE is parallel to BC. Find DE.

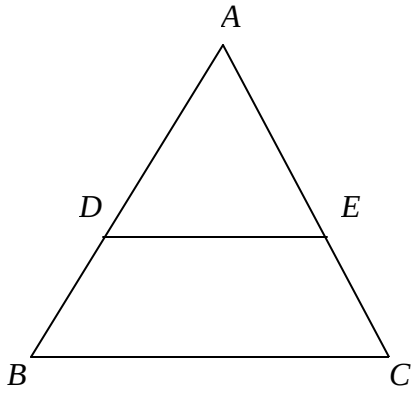
A. $5/3$

B. $21/4$

C. 5

D. $21/8$

E. $3/7$



17. Let AC be a diameter of the circle O with a radius 5. B is a point on the circle and $BC = 4$. Find AB.

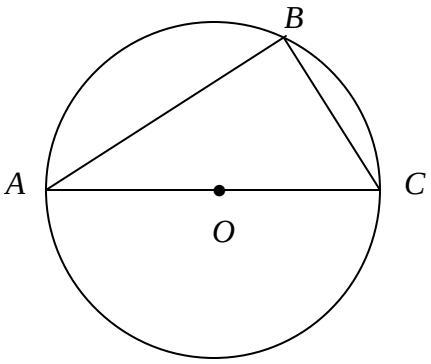
A. 84

B. $2\sqrt{21}$

C. $\sqrt{6}$

D. 42

E. $\sqrt{116}$



18. Let $f(x) = \dfrac{15x-1}{3x^4 - 4x^3 - 2x^2 + x}$. What is the number of vertical asymptotes with equation $x = a$ $(a > 0)$ of $f(x)$?

A. 4

B. 3

C. 2

D. 1

E. 0

19. If $x+y+z=10$, $x^2+y^2+z^2=16$, and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=21$, find xyz.

A. 2

B. 4

C. $\frac{1}{2}$

D. $\frac{1}{4}$

E. 0

20. What is the surface area of a regular tetrahedron having edge length of 4?

A. 24

B. 18

C. $16\sqrt{3}$

D. $18\sqrt{3}$

E. 64

21. (**Tie Break No. 3**) Let $f(x)$ be a real-valued function with inverse given by $f^{-1}(x)=x(4+x^2)+2(6+x^2)$. What is the value of $f\left(f^{-1}\left(f(4)\right)\right)$?

A. 4

B. -4

C. 124

D. -2

E. -124

22. Let $f(x) = (x-a)(x-b)$, where a and b are real numbers. If $f(x)$ has a minimum on the interval $(-1, 1)$, which of the following must be true about a and b?

A. $-1 < ab < 1$

B. $-1 < a + b < 1$

C. $-1 < \frac{ab}{2} < 1$

D. $-1 < \frac{a+b}{2} < 1$

E. None of the above

23. Three circles have the same radius r. Each circle's center is on the two other circles. Find the area of the region common to all three circles.

A. $\frac{p - \sqrt{3}}{2} r^2$

B. $\frac{p - \sqrt{3}}{3} r^2$

C. $\frac{p - \sqrt{2}}{2} r^2$

D. $\frac{p - \sqrt{2}}{3} r^2$

E. None of the Above

24. Solve the equation $3^{2x} + 3^{x+1} - 4 = 0$ in the real number system.

A. $\ln 3$

B. $\log_3 2$

C. 3

D. 0

E. None of the above

25. The graph of which of the following trig functions has amplitude 2 and period $4p$?

A. $y = 2\sin(2x)$

B. $y = \sin(2x - 4)$

C. $y = 2\tan\left(\frac{1}{2}x\right)$

D. $y = -2\cos\left(\frac{1}{2}x\right)$

E. $y = 2\sin(2x + 4)$

26. The graph of function $y = f(x)$ is symmetric to the graph of $y = e^{3x}$ with respect to the straight line $y = x$. Which of the following statements is true?

A. $f(3x) = e^{3x}$

B. $f(3x) = \frac{1}{3}\ln 3 \cdot \ln x$

C. $f(3x) = \ln 3x$

D. $f(3x) = \ln 3 + \ln x$

E. $f(3x) = \ln \sqrt[3]{3} + \ln \sqrt[3]{x}$

27. Given $f(x+1) = x^2 - 3x + 2 \ \ (-\infty < x < 3/2)$, find the domain of the function $y = f^{-1}(x)$.

A. $(-\infty, 5/2)$

B. $(5/2, \infty)$

C. $(-\infty, -1/4)$

D. $(-1/4, \infty)$

E. $(-5/2, \infty)$

28. For what real values of x is $\left|x^2+3x+2\right| = x^2+3x+2$?

A. all real numbers

B. $x \le -2$ or $-1 \le x$

C. $-2 < x < -1$

D. $0 \le x$

E. None of the above

29. Find the domain of the composite function $f \circ g$ where $f(x) = \dfrac{5}{x+5}$ and $g(x) = -\dfrac{5}{x}$.

A. $\{x \mid x \neq 0\}$

B. $\{x \mid x \neq -5\}$

C. $\{x \mid x \neq 0, x \neq -5\}$

D. $\{x \mid x \neq 0, x \neq 1\}$

E. None of the above.

30. Which of the following is NOT a true statement for all real j ?

A. $\sin(-j) = -\sin(j)$

B. $\tan\left(\dfrac{p}{2} - j\right) = \cot j$

C. $1 + \cot^2 j = \sec^2 j$

D. $\sin^2 j + \cos^2 j = 1$

E. $\cos j = \dfrac{1}{\sec(-j)}$

31. Find the following limit where *n* is an integer: $\lim_{n\to\infty} \frac{1+2+3+\cdots+2n}{n^2+1}$

A. 2

B. 1

C. $1/2$

D. $2/3$

E. $3/4$

32. (**Tie Break No.4**) Find a vertical line $x = k$ that divides the area enclosed by $x = \sqrt{y}$, $x = 2$, and $y = 0$ into two equal parts.

A. $x = \sqrt[3]{4}$

B. $x = \frac{3}{2}$

C. $x = \sqrt{2}$

D. $x = 1$

E. $x = \frac{3\sqrt{2}}{2}$

33. Find $\frac{d}{dx}[f(x)]$ if $\frac{d}{dx}[f(3x)] = 6x$.

A. $f'(x) = 3$

B. $f'(x) = 6$

C. $f'(x) = 2x$

D. $f'(x) = \frac{1}{2}x$

E. $f'(x) = \frac{2}{3}x$

34. Express $\frac{\sin x}{1-\cos x} - \frac{\sin x}{1+\cos x}$ in terms of a single trigonometric function.

A. $2\sin x$

B. $2\cos x$

C. $2\tan x$

D. $2\cot x$

E. None of the above

35. Find the distance from a highest point to its closest lowest point on the graph of $y = \cos x$.

A. 1

B. 2

C. $p + 2$

D. $2\sqrt{p^2+1}$

E. $\sqrt{p^2+4}$

36. The two shorter sides of a right triangle have lengths 2 and $\sqrt{5}$. Let x be the smallest angle of the triangle. What is $\cos x$?

A. $\frac{\sqrt{5}}{9}$

B. $\frac{2}{9}$

C. $\frac{2}{3}$

D. $\frac{\sqrt{5}}{3}$

E. $\frac{2}{\sqrt{5}}$

37. If we have $f(\sin x) = 5 - \cos 4x$, then $f(\cos x) = ?$

A. $5 - \cos 4x$

B. $5 + \cos 4x$

C. $5 + \sin 4x$

D. $5 - \sin 4x$

E. None of the above

38. (**Tie Break No. 5**) Solve the inequality $0 < \sin x + \cos x < 1$ for $0 \le x < 2p$.

A. $(0, 3p/4) \cup (7p/4, 2p)$

B. $(2p/3, 3p/4) \cup (11p/6, 2p)$

C. $(p/2, 3p/4) \cup (7p/4, 2p)$

D. $(2p/3, 3p/4) \cup (11p/6, 2p)$

E. None of the above

39. $\sin\left(\frac{p}{12}\right) = ?$

A. $\frac{\sqrt{6} - \sqrt{2}}{4}$

B. $\frac{\sqrt{6} + \sqrt{2}}{4}$

C. $\frac{4}{\sqrt{6} + \sqrt{2}}$

D. $\frac{1}{4}$

E. None of the Above

40. Using degree measure, evaluate the following sum.
$\sin^{-1}(\sin 100^\circ) + \cos^{-1}(\cos 100^\circ) + \tan^{-1}(\tan 100^\circ)$

A. 100°

B. 200°

C. 260°

D. 300°

E. None of the above

41. A bowling ball with a circumference of 30 inches is tightly packed into the smallest possible cubical box for shipment. What proportion of the box's space is occupied by the bowling ball?

A. $\frac{1}{6}$

B. $\frac{p}{6}$

C. $\frac{1}{6p}$

D. $\frac{2}{3}$

E. $\frac{2p}{3}$

42. There are 1817 freshmen at Mullet University this semester. Of these, 458 are taking at least one course in computer science, 571 are taking at least one course in mathematics, and 300 are taking courses in both computer science and mathematics. How many of these freshmen are not taking a course in either computer science or mathematics this semester?

A. 488

B. 788

C. 1029

D. 1088

E. 1517

43. A right circular cone is inscribed in a sphere with a radius r. If the area of the base of the cone is $\frac{3\pi r^2}{4}$. Find the volume of the cone.

A. $\frac{3\pi r^3}{8}$

B. $\frac{3\pi r^3}{16}$

C. $\frac{9\pi r^3}{16}$

D. $\frac{5\pi r^3}{12}$

E. None of the above

44. Find the oblique asymptote of $H(t) = \frac{2t^3 + 11t^2 + 5t - 1}{t^2 + 6t + 5}$.

A. $y = 2t$

B. $y = 2t - 1$

C. $y = 2t + 1$

D. $y = 0$

E. The graph of $H(t)$ does not have an oblique asymptote.

45. If $\cos j = 5/13$ and $270^\circ < j < 360^\circ$, find $\tan j$.

A. $5/12$

B. $-12/5$

C. $12/13$

D. $-12/13$

E. $-5/12$