

CIPHERING

1. A tree casts a shadow 21 meters long. At the same time, the shadow cast by a nearby 48-cm tall statue is 56 cm long. Find the height of the tree in meters.

Correct Answer: 18 meters

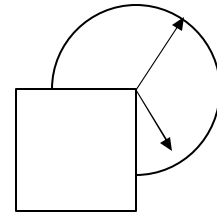
2. A rectangle has sides 4 and 8. A parallelogram has the same area as the rectangle and each side is equal to one of the diagonals of the rectangle. Find the sine of the acute angle q between the sides of the parallelogram.

Correct Answer: $\frac{2}{5}$

3. Rob paints $\frac{1}{4}$ of a barn and Roy paints ? of the remainder. What fraction of the barn is left unpainted?

Correct Answer: $\frac{21}{32}$

4. As illustrated, a sticker covers one quarter of the face of a clock. Assume that the thicknesses of both the hands are 0. For what fraction of the day are both hands not completely visible at the same time?
(Write your answer in the reduced form.)



Correct Answer: $\frac{7}{16}$

5. Simplify $\left(\frac{1-i}{1+i}\right)^2$

Correct Answer: -1

6. Probability of winning a bet is 70%. What are the odds against winning the bet?

Correct Answer: 3:7

7. $\sin(2\sin^{-1}x) = ?$

Correct Answer: $2x\sqrt{1-x^2}$

8. Find the value of n for which the coefficients of the fifth and eighth terms in the expansion of $(x+y)^n$ are the same.

Correct Answer: $n = 11$

9. If $x = \log_5\left(\frac{15(125)}{(75)(625)}\right)$, what is the value of x ?

Correct Answer: -2

10. Who am I?



Hint: My last words are "Do not disturb my circles."

Correct Answer: Archimedes

11. If the equation $x^2 - bx + 4 = 0$ has no real solutions, find all possible values of the parameter b .

Correct Answer: $(-4, 4)$ or $-4 < b < 4$

12. If the range of $y = f(x)$ is $[-5, \infty)$, what is the range of $y = -|f(x)|$? Write your answer in interval notation.

Correct Answer: $(-\infty, 0]$

13. The sum of first n terms of an arithmetic sequence is 143. If the first term is 24 and the n^{th} term is -2 , find n , the total number of terms in the sequence.

Correct Answer: $n = 13$

14. Both m and the complex number $z = (m^2 + i)(1 + mi)$ are real numbers. Find m

Correct Answer: -1

15. Given that $f(x) = \log_b(x)$ and $f(3) = 2$, then $f^{-1}(-2) = ?$

Correct Answer: $\frac{1}{3}$

16. The functions f and g are completely defined by the following table:

x	1	2	3
$f(x)$	1	3	1
$g(x)$	3	2	1

What x -value satisfies $f(g(x)) > g(f(x))$?

Correct Answer: $x = 2$

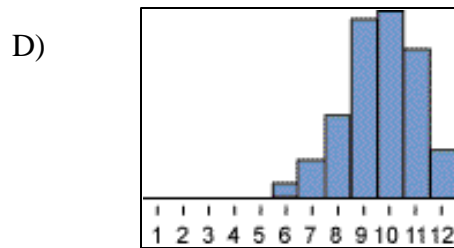
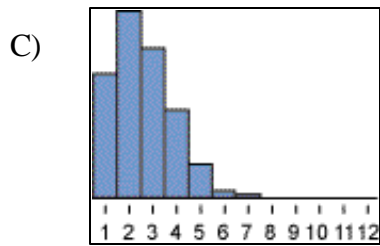
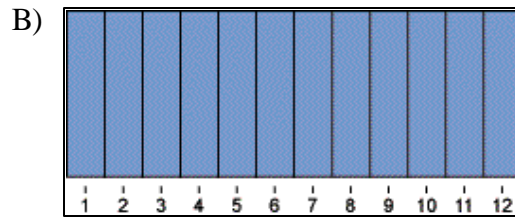
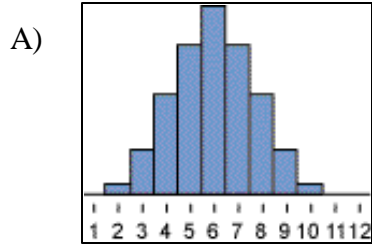
17. If $\csc t = \sqrt{5}$ and t terminates in QII, find $\sec 2t$.

Correct Answer: $\frac{5}{3}$

18. If $a_1 = \frac{1}{3}$ and $a_{m+n} = a_m \cdot a_n$ for any natural numbers m and n , find $\sum_{i=1}^{\infty} a_i = ?$

Correct Answer: $\frac{1}{2}$

19. In which of the following distributions is the mean less than the median?



Correct Answer: D

20. Who am I?



Hint: "The Prince of Mathematicians"

Correct Answer: Carl Friedrich Gauss