23rd ANNUAL

NORTHWEST FLORIDA STATE COLLEGE

MATHEMATICS TOURNAMENT

WRITTEN TEST

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Test Booklet

INSTRUCTIONS: **This is a 90 minute, 45 problem, multiple-choice examination.** There are five (5) possible responses to each question or problem. You are to **select the one (1) best answer to each.** You may mark on the test booklet, and the back of each page may be used for additional work space. Darken completely the circle below the letter of your response to each question on your score sheet. Your student number is encoded on your score sheet for you. Mark your answers **boldly** with a No. 2 pencil. If you must change an answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. Do not mark beyond question 45. Your score will be computed by the following formula: Score = 45 + (4C - I) , where C represents the number of correct answers and I represents the number of incorrect answers. If you can definitely rule out at least one choice, it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. In the event of a tie, the indicated tie-breaker questions will be checked in order until the tie is broken.

Review and check your score sheet carefully. Your student identification number has been encoded on your answer sheet and it has been checked by our marked-sense card reader. If you alter this number in any way you may **disqualify yourself and your team from consideration for any awards.**

When you complete your test, close your test booklet, turn your answer sheet over, and sit quietly until all of the answer sheets are collected. You may keep your pencil and your test booklet. **Calculators are Not Allowed!**

PLEASE DO NOT OPEN
UNTIL INSTRUCTED TO DO SO

1. The sum of the squares of two positive numbers is 185. If one of the numbers is 3 greater than the other, find the greater number.

A. 8

B. 5

C. 9

D. 11

E. 7

2. The base of an isosceles triangle exceeds each of the equal sides by 8 feet. If the perimeter is 89 feet, what is the length of the base in feet?

A. 27

B. 29

C. 35

D. 54

E. 70

3. If # is a binary operation such that $a\#b$ is defined as $\left(\frac{a^2+b^2}{a^2-b^2}\right)$ and $\left(a^2-b^2\neq 0\right)$, then what is the value of $a\#b$ if $2a=b$ and $a\neq 0$?

A. $4/3$

B. $3/5$

C. $-1/2$

D. $-3/5$

E. $-5/3$

4. The domain of the real-valued function $f(x,y)=\sqrt{x^2-y^2-4}+\sqrt{x+y}$ is symmetric with respect to which of the following?

A. the x-axis

B. the y-axis

C. the line $y=x$

D. line $y=-x$

E. the origin

5. Determine the parameter m so that the binomial $x-1$ is a factor of the polynomial x^3-3x^2+mx-4.

A. 4

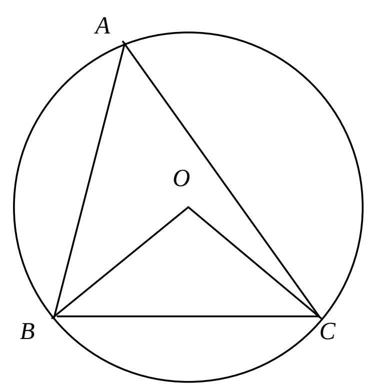
B. 6

C. 2

D. 0

E. -2

6. In the figure, ΔABC is inscribed in a circle with center O. If $\angle BOC=100^\circ$ and $\angle ABO=25^\circ$, then $\angle ACO=?$



A. 15°

B. 50°

C. 40°

D. 30°

E. 25°

7. **(Tie Break No. 1)** In the figure, ΔABC is inscribed in a semicircle whose diameter is $AC = 17$. If two semicircles are drawn with diameters *AB* and *BC*, with $AB = 8$ and $BC = 15$, find the area of the shaded region.

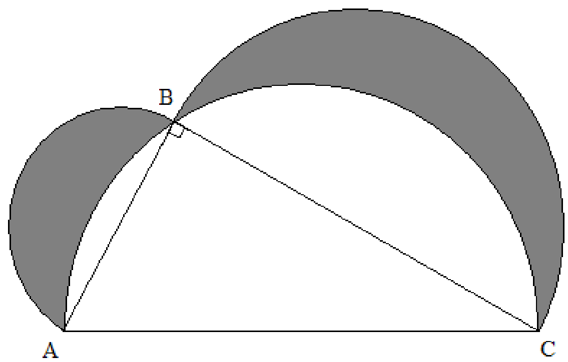
A. 51

B. 60

C. 64

D. 68

E. None of the above



8. If $f(n) = \begin{cases} n^2 & n\, is\, odd \\ -n^2 & n\, is\, even \end{cases}$ and $a_n = f(n) + f(n+1)$, find the sum $S_{100} = \sum_{n=1}^{100} a_n$.

A. 100

B. 150

C. 200

D. 60

E. 20

9. A circle passes though the points $(0, 0)$ and $(2, 4)$. Its center is on the line $2x - y = 5$. Find the equation of the circle.

A. $x^2 + y^2 - 4x - 3y = 0$

B. $x^2 + y^2 - 6x - 2y = 0$

C. $x^2 + y^2 - 2x - 4y = 0$

D. $x^2 + y^2 + 2x - 6y = 0$

E. $x^2 + y^2 + 4x - 7y = 0$

10. Marbles are to be drawn at random without replacement from a bag containing only black and white marbles. If two marbles are randomly drawn, the probability that they are both white is 1/3. If, however, three marbles are randomly drawn from this bag, the probability that all three are white is 1/6. How many marbles are in the bag?

A. 8

B. 10

C. 12

D. 16

E. 18

11. $\frac{2x^3 - 23x^2 + 85x - 95}{(x-4)^4}$ is equal to which of the following expressions?

A. $\frac{5}{(x-4)^4} + \frac{3}{(x-4)^3} - \frac{1}{(x-4)^2} + \frac{2}{(x-4)}$

B. $\frac{5}{(x-4)^4} - \frac{3}{(x-4)^3} + \frac{1}{(x-4)^2} + \frac{2}{(x-4)}$

C. $\frac{5}{(x-4)^4} + \frac{1}{(x-4)^3} - \frac{3}{(x-4)^2} + \frac{2}{(x-4)}$

D. $\frac{2}{(x-4)^4} + \frac{1}{(x-4)^3} - \frac{3}{(x-4)^2} + \frac{5}{(x-4)}$

E. None of the above

12. If m is an integer such that $-5 < m < 2$, and n is an integer such that $-4 < n < 5$, what is the least possible value for $3m^2 - 2n$?

A. -85

B. -75

C. -10

D. -8

E. 0

13. The number of bacteria in a certain culture is related to time by $N = N_0 e^{-0.5t}$, where N_0 is the amount present initially and t is elapsed time in hours. If the initial number of bacteria was 500,000, how much time would elapse for the number of bacteria to be 100,000?

A. $t = 2\ln 0.2$

B. $t = 2\ln 0.5$

C. $t = 2\ln 2$

D. $t = 2\ln 5$

E. None of the above

14. Find the solution set for the system $\begin{cases} \dfrac{2}{x} + \dfrac{4}{y} = 1 \\ y - 4x = 0 \end{cases}$

A. $(4,1)$

B. $(3,12)$

C. $(1,-4)$

D. $(2,8)$

E. None of the above

15. If $\begin{vmatrix} 1 & -2 & 3 \\ 4 & 1 & -2 \\ 8 & x & 6 \end{vmatrix} = -8$, solve for *x*.

A. -5

B. 3

C. -3

D. 8

E. -8

16. For a triangle, the differences between the semi-perimeter and the sides are 3, 4, and 5 respectively. What is the area of the triangle?

A. $6\sqrt{5}$

B. 6

C. $12\sqrt{5}$

D. $18\sqrt{5}$

E. 12

17. **(Tie Break No. 2)** In the figure, the length of circle's radius is 3 and its center is at *O*. *AC* is tangent to the circle at *C*. $AC = 2\sqrt{3}$ and $AB = 6$. Find the distance from *O* to *AB*.

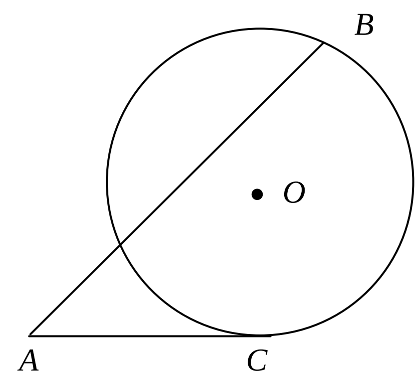
A. $\sqrt{7}/2$

B. 2

C. $\sqrt{3}$

D. 1

E. $\sqrt{5}$



18. Find the domain of the function $f(x) = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{x}}}$

A. $\{x \mid x \neq 0\}$

B. $\{x \mid x \neq -1,\ x \neq 0\}$

C. $\left\{x \mid x \neq -1,\ x \neq -\frac{1}{2},\ x \neq 0\right\}$

D. $\left\{x \mid x \neq -1,\ x \neq -\frac{2}{3},\ x \neq -\frac{1}{2},\ x \neq 0\right\}$

E. $\left\{x \mid x \neq -1,\ x \neq -\frac{2}{3},\ x \neq 0\right\}$

19. $\sin\left(\frac{7\pi}{12}\right) = ?$

A. $\frac{\sqrt{6}-\sqrt{2}}{4}$

B. $\frac{\sqrt{2}-\sqrt{6}}{4}$

C. $\frac{\sqrt{6}+\sqrt{2}}{4}$

D. $\frac{\sqrt{6}+\sqrt{2}}{2}$

E. $\frac{\sqrt{6}-\sqrt{2}}{2}$

20. Jane is 6 years older than Tom, and Tom is 5 years younger than Phillip. Chris is 3 years older than Tom. If Jane's age is expressed as *J*, what is the sum of the ages of Jane, Tom, Phillip, and Chris in terms of *J*?

A. $4J - 10$

B. $J - 9$

C. $3J - 6$

D. $4J + 12$

E. $J + 14$

21. Solve the inequality $0.2(x^2 - 5x + 4)(x^2 - 24x + 144)(x - 20) \geq 0$

A. $(-\infty, 1] \cup [4, 12]$

B. $[1, 4] \cup [20, \infty)$

C. $(-\infty, 1] \cup \{4\} \cup [12, 20]$

D. $[1, 4] \cup \{12\} \cup [20, \infty)$

E. $(-\infty, 1] \cup \{12\} \cup [20, \infty)$

22. Suppose $f(x)$ is a function such that $f(x) = f^{-1}(x)$, and $f(0) = -\frac{1}{3}$. Evaluate

$$f^{-1}\left(\frac{1}{3} + f\left(\frac{1}{3} + f^{-1}\left(\frac{1}{3} + f(0)\right)\right)\right)$$

A. 0

B. $\frac{1}{27}$

C. $-\frac{1}{81}$

D. $-\frac{1}{3}$

E. 1

23. Multiply and simplify $\sqrt[3]{3} \cdot \sqrt{10}$

A. $\sqrt[6]{30}$

B. $\sqrt[5]{30}$

C. $\sqrt[6]{9000}$

D. $\sqrt[5]{9000}$

E. $\sqrt[4]{30}$

24. **(Tie Break No. 3)** In a trapezoid *ABCD*, $AD // BC$. The diagonals *AC* and *BD* intersect at point *O*. The area of ΔAOD is 5 and the area ΔBOC is 9. Find the area of the trapezoid.

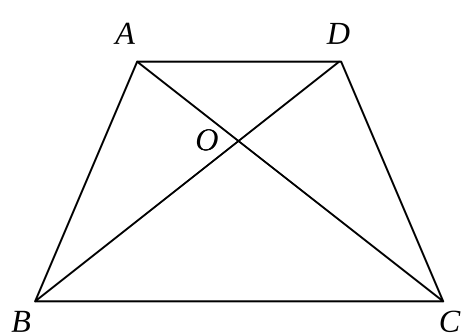
A. 28

B. $24\sqrt{5}$

C. $20\sqrt{5}$

D. 26

E. None of the above



25. Determine the constant term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$.

A. 144

B. 288

C. 495

D. 11880

E. None of the above

26. In the figure, the sides of rectangle *ABCD* have length 8. If it is folded along the dashed line *MN*, the vertex *B* is positioned at the mid-point E of the side *CD* and the vertex *A* is positioned at point *F*. Find the length of *CN*.

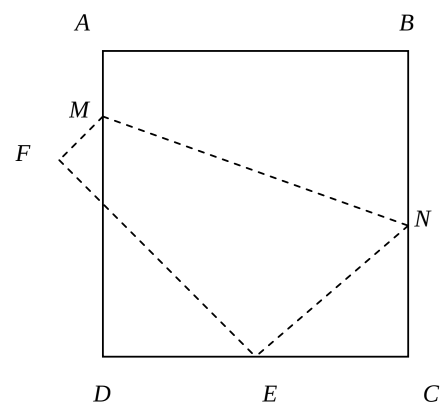
A. 2

B. $\sqrt{5}$

C. 3

D. 4

E. $\sqrt{3}$



27. In ΔABC, $\angle ACB = 90^\circ$, $AC = 3$ and $BC = 4$. D is a point outside of the triangle. If $AD = \sqrt{10}$ and $CD = 5$, which of the following may be the length of BD.

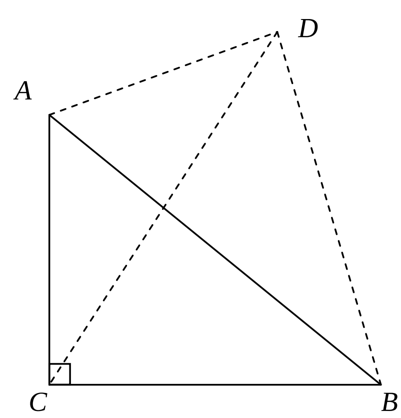
A. $\sqrt{15}$

B. 4

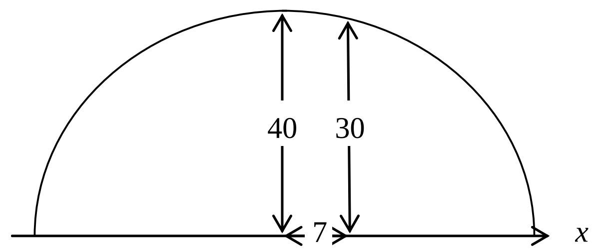
C. $\sqrt{17}$

D. 3

E. None of the above



28. A railroad tunnel is shaped like a semi-ellipse. The height of the tunnel at the center is 40 *ft* and the vertical clearance must be 30 *ft* at a point 7 *ft* from the center. Find an equation for the ellipse



A. $\frac{x^2}{112}+\frac{y^2}{900}=1$

B. $\frac{x^2}{112}+\frac{y^2}{1600}=1$

C. $\frac{x^2}{49}+\frac{y^2}{1600}=1$

D. $\frac{x^2}{1600}+\frac{y^2}{112}=1$

E. $\frac{x^2}{1600}+\frac{y^2}{49}=1$

29. Solve the inequality $|3x+9|<|x-1|$

A. $(-\infty,-5)\cup(-2,\infty)$

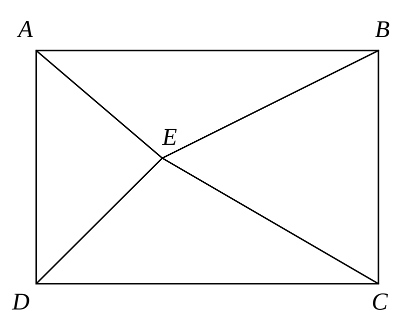
B. $(2,5)$

C. $(-5,-2)$

D. $\varnothing$

E. $(-\infty,\infty)$

30. In the rectangle *ABCD*, ΔAED and ΔBEC are both isosceles with $AE=DE$ and $BE=CE$. If the ratio of the area of ΔAED to the area of ΔBEC is 2:3, find the ratio of the area of ΔAED to the area of ΔAEB.



A. 2:3

B. 3:4

C. 4:5

D. 5:6

E. None of the above

31. Solve $\tan x = 2\sin x$ giving all solutions between $-\pi$ and π, inclusive.

A. $\{-\pi,\ 0,\ \pi\}$

B. $\{-\pi,\ \pi\}$

C. $\left\{-\pi,\ \frac{-\pi}{3},\ \frac{\pi}{3},\ \pi\right\}$

D. $\left\{\frac{-\pi}{3},\ 0,\ \frac{\pi}{3}\right\}$

E. None of the above

32. If $\tan\alpha = 2, \tan\beta = 3$, and $\theta = \frac{\alpha+\beta}{2}$, find the value of $\frac{\sin\theta\cos\theta}{\sin\left(\theta - \frac{\pi}{4}\right)\sin\left(\theta + \frac{\pi}{4}\right)}$

A. 2

B. $\frac{1}{5}$

C. $\sqrt{5}$

D. 1

E. $\frac{3\sqrt{2}}{4}$

33. A sequence $\{a_n\}$ is defined by $a_0 = 7$, $a_1 = 3$, and $3a_n = 2a_{n-1} + a_{n-2}$ $(n \geq 2)$. Find $\lim_{n\to\infty} a_n$.

A. $2/3$

B. $5/2$

C. 3

D. 4

E. 5

34. The surface area of a large spherical balloon is doubled. By what factor is the volume increased?

A. 3

B. $2\sqrt{2}$

C. 8

D. 4

E. 6

35. In a 110-volt circuit having a resistance of 11 ohms, the power, W (Watts) is given by $W = 110I - 11I^2$ when a current, I (Amps), is flowing. Determine the maximum power that can be delivered in this circuit?

A. 220 Watts

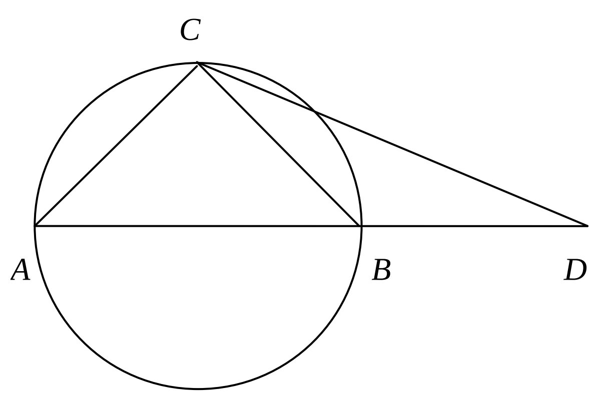
B. 200 Watts

C. 275 Watts

D. 230 Watts

E. 225 Watts

36. Triangles ΔABC and ΔCBD are each isosceles with $AC = BC = BD$. If ΔABC is inscribed in a circle with radius 1 and AB is a diameter of the circle, find CD.



A. $2+\sqrt{2}$

B. $1+2\sqrt{2}$

C. $\sqrt{2+\sqrt{2}}$

D. $\sqrt{4+2\sqrt{2}}$

E. $2\sqrt{2}$

37. If the angle θ is terminated in the first quadrant and $\cos\theta = \frac{3}{5}$, find the value of $\tan\theta$.

A. $5/3$

B. $4/5$

C. $3/5$

D. $3/4$

E. $4/3$

38. If $f(x) = 2a\sin x\cos x + 2b\cos^2 x$, $f\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{2} + 6$, and $f(0) = 8$, find the maximum value of $f(x)$.

A. 8

B. 7

C. 9

D. 10

E. 12

39. **(Tie Break No. 4)** If $\tan\frac{\alpha+\beta}{2} = \frac{\sqrt{6}}{2}$ and $\tan\alpha\tan\beta = \frac{13}{7}$, find the value of $\cos(\alpha-\beta)$.

A. $3/4$

B. $2/3$

C. $\sqrt{2}/4$

D. $\sqrt{2}/3$

E. $1/3$

40. Find the value of $\sin^2 20^\circ + \cos^2 50^\circ + \sin 20^\circ \cos 50^\circ$

A. $3/4$

B. $2/3$

C. $\sqrt{2}/4$

D. $\sqrt{2}/3$

E. $1/3$

41. Find the exact value of $\tan\frac{\pi}{8}$

A. $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

B. $\frac{1}{\sqrt{2}-1}$

C. $\sqrt{2}+1$

D. $\sqrt{2}-1$

E. None of the above

42. An airplane leaves an aircraft carrier and flies due south at 400 km/hr. The carrier is heading in the direction North 60° West of at 32 km/hr. If the plane has 5 hours of fuel, what is the maximum distance south the plane can travel so that the fuel remaining will allow a safe return to the carrier at 400 km/hr?

A. 955 km

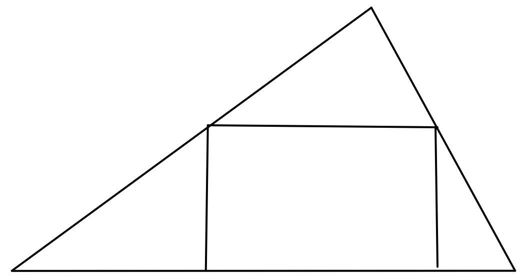
B. 806 km

C. 828 km

D. 917 km

E. 945 km

43. A rectangle is inscribed in a right triangle as shown in the accompanying figure. If the three sides of the triangle have the lengths 6, 8, and 10 respectively, find the maximum area of the rectangle.



A. 8

B. 12

C. 15

D. 18

E. None of the above

44. Find $A+B$, where A and B are integers satisfying the expression $\sin 3\theta = A\sin\theta + B\sin^3\theta$

A. -1

B. -4

C. 0

D. 1

E. 4

45. **(Tie Break No. 5)** Ali and Bill play a game in which they alternately toss a pair of fair, six-sided dice. The first one to get a total of 7 on a single toss of the dice wins the game. Find the probability that the player who tosses first will win the game.

A. $\frac{1}{6}$

B. $\frac{1}{2}$

C. $\frac{5}{16}$

D. $\frac{6}{11}$

E. None of the above