Ciphering

1. If $f(x) = x^2 + |x+1|$, then $f(-2) = ?$

$f(-2) = (-2)^2 + |-2+1| = 4+1 = 5$

Correct Answer: 5

2. What is the numerical value of $\sum_{n=1}^{\infty} \frac{2^{n+2}}{3^n}$?

$$\sum_{n=1}^{\infty} \frac{2^{n+2}}{3^n} = \sum_{n=1}^{\infty} \frac{8}{3}\left(\frac{2}{3}\right)^{n-1} = \frac{8}{3} \cdot \frac{1}{1-\frac{2}{3}} = \frac{8}{3} \cdot \frac{1}{\frac{1}{3}} = 8$$

Correct Answer: 8

3. You are given $P(A \cup B) = 0.7$ and $P(A \cup B') = 0.9$. Determine $P(A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7$ (1)
$P(A \cup B') = P(A) + P(B') - P(A \cap B') = 0.9$ (2)
Add (1) and (2)
$2P(A) + P(B) + P(B') - P(A \cap B) - P(A \cap B') = 1.6$
$2P(A) + 1 - P(A) = 1.6$
$P(A) = 0.6$

Correct Answer: $P(A) = 0.6$

4. What is the area of a square whose diagonal is one unit longer than the length of its side?

Let x be the length of its side. $x^2 + x^2 = (x+1)^2$
$x^2 - 2x - 1 = 0$, $x = 1 + \sqrt{2}$, $x^2 = \left(1+\sqrt{2}\right)^2 = 3 + 2\sqrt{2}$

Correct Answer: $3 + 2\sqrt{2}$

5. Find the sum of the series $\frac{1}{3}-\frac{1}{2}+\frac{1}{9}-\frac{1}{4}+\frac{1}{27}-\frac{1}{8}+\cdots\cdots$

$$\frac{1}{3}-\frac{1}{2}+\frac{1}{9}-\frac{1}{4}+\frac{1}{27}-\frac{1}{8}+\cdots\cdots=\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots\right)-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\right)$$
$$=\frac{1}{3}\cdot\frac{1}{1-\frac{1}{3}}-\frac{1}{2}\cdot\frac{1}{1-\frac{1}{2}}=\frac{1}{2}-1=-\frac{1}{2}$$

Correct Answer: $-\frac{1}{2}$

6. If $\sin\left(\frac{\pi}{2}+\theta\right)=\frac{3}{5}$, find $\cos 2\theta$.

$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta=\frac{3}{5}$ and $\cos 2\theta=2\cos^2\theta-1=2\left(\frac{3}{5}\right)^2-1=-\frac{7}{25}$

Correct Answer: $-\frac{7}{25}$

7. A piece of coaxial cable $11\frac{2}{3}$ feet long is to be divided into two parts such that one part is $\frac{2}{3}$ the length of the other. What is the length of the shorter piece?

Let *x* be the length of the shorter piece.
$x+\frac{3}{2}x=\frac{35}{3}$, $\frac{5}{2}x=\frac{35}{3}$, $x=\frac{14}{3}$ or $x=4\frac{2}{3}$

Correct Answer: $4\frac{2}{3}$ feet

8. Solve for *x*: $9^{2x}\cdot 27^{x^2}=3^{-1}$

$\left(3^2\right)^{2x}\cdot\left(3^3\right)^{x^2}=3^{-1}$, $3^{4x}\cdot 3^{3x^2}=3^{-1}$, $3^{3x^2+4x}=3^{-1}$, $3x^2+4x+1=0$,
$(3x+1)(x+1)=0$, $x=-1$ or $x=-\frac{1}{3}$

Correct Answer: $x=-1$ or $x=-\frac{1}{3}$

9. The following table gives values of f, f', g, and g' at selected values of x. If $h(x) = f(g(x))$, then $h'(1) = ?$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

$h'(1) = f'(g(1)) \cdot g'(1) = f'(-1) \cdot 2 = 5 \cdot 2 = 10$

Correct Answer: 10

10. Who am I?

Hint: I developed a non-Euclidean geometry.

Correct Answer: Nikolai Lobachevsky

11. If the length of a rectangle is increased by 30% while its width is decreased by 20%, give the percent of increase/decrease in the area.

Let l be the length of the rectangle and w the width. The ratio of the area change is $\frac{(1.3l)\cdot(0.8w)}{lw} = 1.04$. The area increases 4%.

Correct Answer: 4%

12. The seventh term of an arithmetic sequence is 41 and the thirteenth term is 77. What is the twentieth term in this arithmetic sequence?

$$\begin{cases} a_7 = a_1 + 6d = 41 \\ a_{13} = a_1 + 12d = 77 \end{cases}$$

$d = 6$ and $a_1 = 5$

$a_{20} = 5 + 19 \cdot 6 = 119$

Correct Answer: 119

13. $\sin\left(2\cos^{-1}\frac{1}{\sqrt{5}}\right) = ?$

Let $\theta = \cos^{-1}\frac{1}{\sqrt{5}}$. Then $\cos\theta = \frac{1}{\sqrt{5}}$ and $\sin\theta = \frac{2}{\sqrt{5}}$.

$$\sin\left(2\cos^{-1}\frac{1}{\sqrt{5}}\right) = \sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{2}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}} = \frac{4}{5}$$

Correct Answer: $\frac{4}{5}$

14. $(1+i)^{12} = ?$

$$(1+i)^{12} = \left((1+i)^2\right)^6 = (2i)^6 = 2^6 i^6 = -64$$

Correct Answer: -64

15. Al can complete a piece of work in 10 days. After he has worked 2 days, Bill comes to help him and together they complete the work in 3 days. In how many days could Bill have completed the original job working alone?

Let *x* be the days needed for Bill to complete the original job working alone.

$2\cdot\frac{1}{10} + 3\left(\frac{1}{10} + \frac{1}{x}\right) = 1$, $\frac{1}{2} + \frac{3}{x} = 1$, $\frac{3}{x} = \frac{1}{2}$, and $x = 6$.

Correct Answer: 6 days

16. Find the maximum value of the trigonometric function $y = \sin\theta + \cos\theta$.

$$y = \sin\theta + \cos\theta = \sqrt{2}\left(\cos\frac{\pi}{4}\sin\theta + \sin\frac{\pi}{4}\cos\theta\right) = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) \le \sqrt{2}$$

Correct Answer: $\sqrt{2}$

17. Determine the ninth term in descending order in the expansion of $\left(x-\frac{1}{\sqrt{x}}\right)^{12}$.

The ninth term is $\binom{12}{8}x^{12-8}\left(-\frac{1}{\sqrt{x}}\right)^{8}=\frac{12!}{8!\cdot 4!}\cdot x^{4}\cdot\frac{1}{x^{4}}=\frac{12\cdot 11\cdot 10\cdot 9}{4\cdot 3\cdot 2\cdot 1}=495$

Correct Answer: 495

18. Simplify $\frac{2^{n+4}-2\left(2^{n}\right)}{2\left(2^{n+3}\right)}$

$\frac{2^{n+4}-2\left(2^{n}\right)}{2\left(2^{n+3}\right)}=\frac{2^{n}\left(2^{4}-2\right)}{2^{n}\cdot 2\left(2^{3}\right)}=\frac{16-2}{16}=\frac{7}{8}$

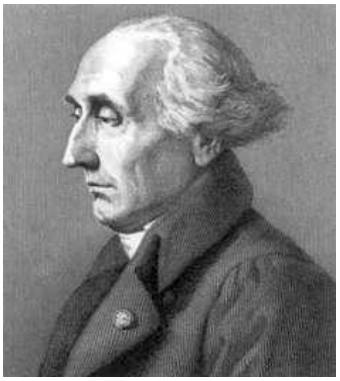
Correct Answer: $\frac{7}{8}$

19. Determine the value of the expression $\log\left(\frac{a^{2}}{bc}\right)+\log\left(\frac{b^{2}}{ca}\right)+\log\left(\frac{c^{2}}{ab}\right)$

$\log\left(\frac{a^{2}}{bc}\right)+\log\left(\frac{b^{2}}{ca}\right)+\log\left(\frac{c^{2}}{ab}\right)=\log\left(\frac{a^{2}b^{2}c^{2}}{bccaab}\right)=\log 1=0$

Correct Answer: 0

20. Who am I?



Hint: The multiplier method in mathematical optimization is named after me.

Correct Answer: Joseph Louis Lagrange