24th ANNUAL

NORTHWEST FLORIDA STATE COLLEGE

MATHEMATICS TOURNAMENT

WRITTEN TEST

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Niceville, Florida

Test Booklet

INSTRUCTIONS: **This is a 90 minute, 45 problem, multiple-choice examination.** There are five (5) possible responses to each question or problem. You are to **select the one (1) best answer to each.** You may mark on the test booklet, and the back of each page may be used for additional work space. Darken completely the circle below the letter of your response to each question on your score sheet. Your student number is encoded on your score sheet for you. Mark your answers **boldly** with a No. 2 pencil. If you must change an answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. Do not mark beyond question 45. Your score will be computed by the following formula: Score = 45 + (4C - I) , where C represents the number of correct answers and I represents the number of incorrect answers. If you can definitely rule out at least one choice, it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. In the event of a tie, the indicated tie-breaker questions will be checked in order until the tie is broken.

Review and check your score sheet carefully. Your student identification number has been encoded on your answer sheet and it has been checked by our marked-sense card reader. If you alter this number in any way you may **disqualify yourself and your team from consideration for any awards.**

When you complete your test, close your test booklet, turn your answer sheet over, and sit quietly until all of the answer sheets are collected. You may keep your pencil and your test booklet. **Calculators are Not Allowed!**

PLEASE DO NOT OPEN
UNTIL INSTRUCTED TO DO SO

1. If $3x^2 - 2x + 7 = 0$, then $\left(x - \frac{1}{3}\right)^2 = ?$

A. $20/9$

B. $7/9$

C. $-7/9$

D. $-20/9$

E. $-8/9$

2. Solve the equation $e^{-x^2} = \left(e^x\right)^2 \cdot \frac{1}{e^3}$

A. $\{-3\}$

B. $\{3, 1\}$

C. $\{-3, 1\}$

D. $\{1\}$

E. $\varnothing$

3. A satellite dish is shaped like a paraboloid of revolution. The signals that emanate from a satellite strike the surface of the dish and are reflected to a single point where the receiver is located. If the dish is 8 feet across at its opening and 3 feet deep at its center, at what position should the receiver be placed? That is, where is the focus?

A. $1\frac{1}{3}$ feet above the vertex along its axis of symmetry.

B. 3 feet above the vertex along its axis of symmetry

C. At the vertex

D. $\frac{4}{3}$ inches above the vertex along its axis of symmetry

E. Not enough information to determine a solution

4. Find the coefficient of x^2 in the expansion of $\left(\sqrt{x}+\frac{3}{\sqrt{x}}\right)^8$

A. 9

B. 252

C. 28

D. 84

E. 756

5. In a quadrilateral *ABCD*, $\angle B = 70^\circ$, $\angle C = 90^\circ$, $AB = DB = 10$, and $CD = 5$. Find $\angle A$.

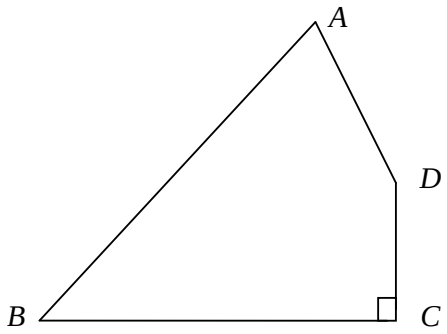
A. 60°

B. 30°

C. 20°

D. 70°

E. 65°



6. Solve for *x*: $\begin{vmatrix} x & 1 & 2 \\ 1 & x & 3 \\ 0 & 1 & 2 \end{vmatrix} = -4x$

A. $\{0, -1/2\}$

B. $\{-1/2\}$

C. $\{0\}$

D. $\{0, 1/2\}$

E. $\{1\}$

7. What is the length of the edge of a cube if its volume could be doubled by an increase of 6 centimeters in one edge, an increase of 12 centimeters in a second edge, and a decrease of 4 centimeters in the third edge?

A. 6 *cm*

B. 36 *cm*

C. 6 *cm* or 12 *cm*

D. 144 *cm*

E. $\sqrt[3]{6}$ *cm*

8. (Tie Break No. 1) $\cos^3 20^\circ + \cos^3 140^\circ + \cos^3 100^\circ = ?$

A. $3/4$

B. $-3/8$

C. $3/8$

D. $-\sqrt{3}/8$

E. $-3/4$

9. Solve the equation $\frac{b+c}{x+a} = \frac{b-c}{x-a}$, where $a \neq 0$, b, and $c \neq 0$ are constants.

A. $x = \frac{ab}{c}$

B. $x = ab$

C. $x = \frac{bc}{a}$

D. $x = ab - c$

E. $x = \frac{ac}{b}$

10. Reversing the digits of Alice's age gives her mother's age. The difference in their ages is 27 years. If the difference of the digits in each age is 3, what is the sum of their ages?

A. 76

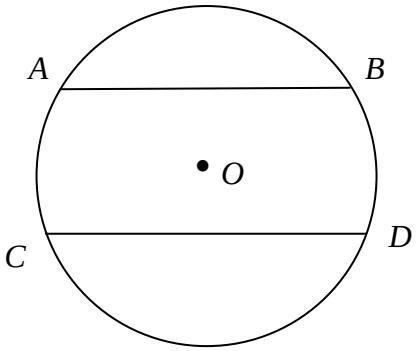
B. 132

C. 87

D. 67

E. 143

11. In the figure, O is the center of the circle. $AB = 40$ and $CD = 48$ are two chords in the circle. $AB // CD$ and the distance between them is 22. Find the radius of the circle.



A. 26

B. 25

C. 27

D. 24

E. $4\sqrt{39}$

12. $\sin^{-1}\left(\cos\frac{9\pi}{7}\right) = ?$

A. $\frac{2\pi}{7}$

B. $\frac{3\pi}{14}$

C. $-\frac{3\pi}{14}$

D. $-\frac{2\pi}{7}$

E. None of the above

13. Solve the equation $\log_5 x + \log_3 x = 1$.

A. $\left\{\dfrac{\ln 5 \cdot \ln 3}{\ln 15}\right\}$

B. $\left\{e^5\right\}$

C. $\left\{e^{\log_{15}(\ln 5 \cdot \ln 3)}\right\}$

D. $\left\{e^{\log_{15} 5^{\ln 3}}\right\}$

E. None of the above

14. The distance between two circles with a common center is 6. If the circumference of the larger circle is 220, find the area of the smaller circle.

A. $\dfrac{12100}{\pi} + 1320 + 36\pi$

B. 208π

C. $\dfrac{12100}{\pi} - 1320 + 36\pi$

D. $\dfrac{12100}{\pi} + 36\pi$

E. $\dfrac{12100}{\pi} - 660 + 36\pi$

15. A circle is inscribed in a trapezoid *ABCD*. $AB // CD$, *E* and *F* are mid-points of *AD* and *BC* respectively. If the perimeter of the trapezoid is 24, find the length of *EF*.

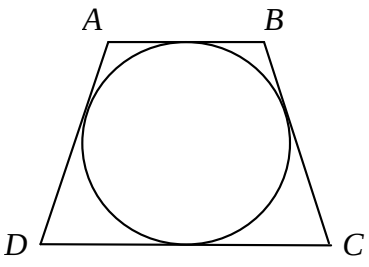
A. 6

B. 7

C. 8

D. 4

E. Not enough information



16. Convert the polar equation $r = \dfrac{6\sec\theta}{2\sec\theta - 1}$ to a rectangular equation.

A. $x^2 + y^2 - 12x - 36 = 0$

B. $4x^2 + 3y^2 - 12x - 36 = 0$

C. $3x^2 + 4y^2 + 12x - 36 = 0$

D. $x^2 + y^2 + 12x + 36 = 0$

E. $3x^2 + 4y^2 - 12x - 36 = 0$

17. (Tie Break No. 2) Two sides of a triangle have lengths 2 units and 4 units, and their included angle measures 165°. Find the area of the triangle.

A. $2\sqrt{2-\sqrt{3}}$

B. $2\left(\sqrt{6}-\sqrt{2}\right)$

C. $\sqrt{6}+\sqrt{2}$

D. $2\sqrt{2+\sqrt{3}}$

E. None of the above

18. Find the value of the sine of an angle between the lines $3x - y + 2 = 0$ and $x + 2y + 1 = 0$.

A. $\dfrac{7}{10}$

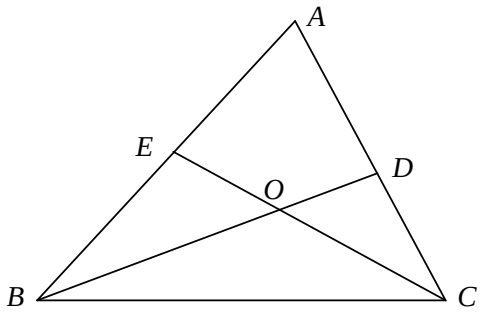
B. $\dfrac{7}{5\sqrt{2}}$

C. $-\dfrac{7}{5\sqrt{2}}$

D. $\dfrac{2}{3}$

E. None of the above

19. In ΔABC, $AB = c$, $BC = a$, and $CA = b$. BD and CE are bisectors of $\angle B$ and $\angle C$, respectively. BD and CE intersect at O. Find the ratio $BO : OD$.



A. $(a+c):b$

B. $(a+b):c$

C. $a:(b+c)$

D. $(b+c):a$

E. $b:(a+c)$

20. $\cos A \cos 2A \cos 4A \cdots \cos(2^{n-1}A)$ is identically equal to which of the following expressions?

A. $\dfrac{\sin(2^n A)}{2^{n-1}\cos A}$

B. $\dfrac{\sin(2^n A)}{2^n \sin A}$

C. $\dfrac{\cos(2^n A)}{2^{n-1}\cos A}$

D. $\dfrac{2^n \sin A}{\sin(2^n A)}$

E. None of the above

21. (Tie Break No. 3) ΔABC is inscribed a circle. If $a = 10$ and $\angle A = 15^\circ$, find the radius of the circle.

A. $10\left(\sqrt{6}-\sqrt{2}\right)$

B. $5\left(\sqrt{6}+\sqrt{2}\right)$

C. $5\left(\sqrt{6}-\sqrt{2}\right)$

D. $10\left(\sqrt{6}+\sqrt{2}\right)$

E. $10\left(\sqrt{3}-1\right)$

22. The sum of the solutions of equation $z - 2|z| = -7 + 4i$ is

A. $\frac{5}{3} + 4i$

B. $3 + \frac{8}{3}i$

C. $\frac{14}{3} + 4i$

D. $\frac{14}{3} + 8i$

E. $8 + \frac{14}{3}i$

23. In the figure, the three sides of ΔABC have the lengths $a = 13$, $b = 14$, and $c = 15$. Find the perimeter of the square inscribed in the triangle.

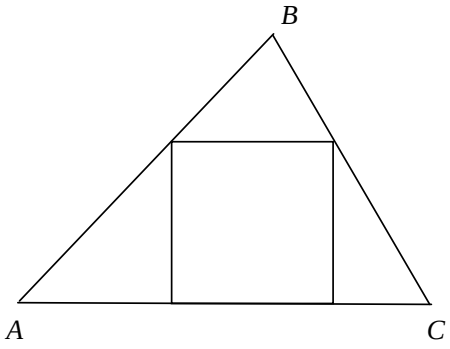
A. $336/13$

B. $84/13$

C. 36

D. $172/7$

E. $364/15$



24. A fifth degree polynomial $p(x)$ with real coefficients has complex zeros $x = 1 \pm i\sqrt{2}$, $x = \pm 3$, and $x = \frac{2}{3}$. If the *y*-intercept of its graph is $(0, 9)$, find the coefficient of its x^4 term.

A. $4/3$

B. $-4/3$

C. -4

D. $-8/3$

E. The answer is not unique.

25. A regular octagon is obtained by cutting four equal right isosceles triangles from four corners of a square. If the length of the sides of the square is 1, find the area of the octagon.

A. $2\left(\sqrt{2}-1\right)$

B. $\dfrac{1}{1+\sqrt{2}}$

C. $\dfrac{1+\sqrt{2}}{2+\sqrt{2}}$

D. $1-\dfrac{\sqrt{2}}{4}$

E. $\sqrt{3}-1$

26. If $\tan\left(\dfrac{\pi}{4}+A\right)=2$, find the value of $2\cos^2 A-\sin 2A$.

A. $5/6$

B. $6/5$

C. $\sqrt{3}/4$

D. $4/\sqrt{3}$

E. $8/7$

27. Which of the following trigonometric expressions is identical to $\sin^2 A-\sin^2 B$?

A. $\cos^2 A-\cos^2 B$

B. $\sin(A+B)\sin(A-B)$

C. $\cos(A+B)\cos(A-B)$

D. $\left(\sin A-\sin B\right)^2$

E. $\left(\cos A-\cos B\right)^2$

28. Find the exact value of $\sin 72^\circ$

A. $\dfrac{\sqrt{5}-1}{4}$

B. $\dfrac{\sqrt{10-2\sqrt{5}}}{4}$

C. $\dfrac{\sqrt{10+2\sqrt{5}}}{4}$

D. $\dfrac{\sqrt{5}+1}{4}$

E. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

29. If $\{a_n\}(a_n > 0)$ is a geometric sequence and $a_5 a_6 = 9$, find the value of the sum $\log_3 a_1 + \log_3 a_2 + \cdots + \log_3 a_{10}$

A. 12

B. 8

C. 10

D. $2 + 10\log_3 2$

E. $2 + \log_3 5$

30. Solve the inequality $2 + \log_{\frac{1}{2}}(5-x) + \log_2 \dfrac{1}{x} > 0$

A. $(4, 5)$

B. $(-\infty, 1)$

C. $(-\infty, 1) \cup (4, \infty)$

D. $(4, \infty)$

E. $(0, 1) \cup (4, 5)$

31. Find the domain of the inverse function of $f(x)=\dfrac{e^x-1}{e^x+1}$.

A. $(-\infty,\ \infty)$

B. $(-1,1)$

C. $(-\infty,-1)\cup(1,\ \infty)$

D. $[-1,1]$

E. $(0,1)$

32. In the figure, AB is a diameter of a semicircle. D and E are two points on the semicircle. AD and BE intersect at the point C. If $AB=16$, $DC=3$ and $EC=2$, find the perimeter of the ΔABC.

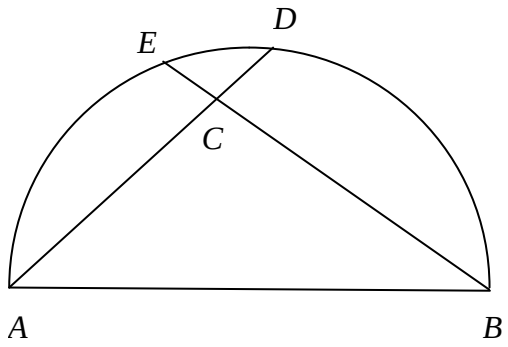
A. 38

B. 36

C. $16\left(\sqrt{2}+1\right)$

D. $12\sqrt{3}+16$

E. 40



33. Write the expression $i^{2011}+i^{2010}+\cdots+i+1$ in standard form $a+bi$.

A. 0

B. $1+i$

C. 1

D. i

E. $1-i$

34. Solve the inequality $\sqrt{2x+3} > x+1$

A. $\left[-\frac{3}{2}, -1\right]$

B. $[-1, \sqrt{2})$

C. $(-\sqrt{2}, \sqrt{2})$

D. $\left[-\frac{3}{2}, \sqrt{2}\right)$

E. $\left[-\frac{3}{2}, -1\right)$

35. If the domain of function $f(x) = (x-1)^{\frac{2}{3}}(ax^2 + 4ax + 3)^{-1}$ is the set of all real numbers, then what are the permissible values of the parameter *a*?

A. $(0, 3/4)$

B. $(-\infty, \infty)$

C. $(3/4, \infty)$

D. $[0, 3/4)$

E. $(-\infty, 0)$

36. $\{a_n\}_{n=1}^{\infty}$ is an arithmetic sequence. $a_7 = 17$ and $a_{31} = 65$. Find $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{a_k a_{k+1}}$.

A. ∞

B. 2

C. $2/5$

D. $1/10$

E. 1

37. Which of the following is the equation of the circle such that its center is on the parabola $y^2 = 2x$ and it is tangent to both the *x*-axis and directrix of the parabola?

A. $x^2 + y^2 - x - 2y - \frac{1}{4} = 0$

B. $x^2 + y^2 + x - 2y + 1 = 0$

C. $x^2 + y^2 - x - 2y + \frac{1}{4} = 0$

D. $x^2 + y^2 - x - 2y + 1 = 0$

E. $x^2 + y^2 - x + 2y - \frac{1}{4} = 0$

38. (Tie Break No. 4) Find the sum of all positive rational numbers that are less than 10 and that have denominator 10 when written in lowest terms.

A. 250

B. 200

C. 180

D. 210

E. 195

39. If the line tangent to the graph of the function $f(x)$ at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

A -5

B. 1

C. 3

D. 7

E. Undefined

40. Urn #1 contains 10 balls: 4 red and 6 blue. Urn #2 contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44. Determine the number of blue balls in urn #2.

A. 4

B. 20

C. 24

D. 44

E. 64

41. Find the real solution of the equation $4^x + 6^x = 9^x$

A. $\ln \dfrac{-1+\sqrt{5}}{2}$

B. $\dfrac{\ln(\sqrt{5}-1) - \ln 2}{\ln 2 - \ln 3}$

C. $\log_{\frac{2}{3}} \dfrac{-1 \pm \sqrt{5}}{2}$

D. $\ln \dfrac{\sqrt{5}-1}{2} - \ln \dfrac{2}{3}$

E. $\dfrac{\ln(\sqrt{5}-1)}{\ln 3}$

42. Find the coefficient of the x^3 term in the expansion of $(1+x)^{10}(1-x)^8$

A. -8

B. -4

C. -16

D. 15

E. -12

43. $\int_a^c f(x)dx = \ln 32$ and $\int_a^c |f(x)|dx = \ln 64$. $f(x) \geq 0$ in (a, b) and $f(x) < 0$ in (b, c). Find the value of $\int_b^c f(x)dx$

A. $-\ln\sqrt{2}$

B. $\ln(1/2)$

C. $-\ln 8$

D. $-\ln 32$

E. $-\ln(1/2)$

44. Which of the following series diverge?

I. $\sum_{n=0}^{\infty}\left(\frac{\sin 2}{\pi}\right)^n$ II. $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt[3]{n}}\right)$ III. $\sum_{n=1}^{\infty}\left(\frac{e^n}{e^n+1}\right)$

A. III only

B. I and II only

C. I and III only

D. II and III only

E. I, II, and III

45. (Tie Break No. 5) $\cot^{-1}\left(\frac{43}{32}\right) - \tan^{-1}\left(\frac{1}{4}\right) = ?$

A. $\tan^{-1}(5/4)$

B. $\cos^{-1}(3/5)$

C. $\sin^{-1}(-3/4)$

D. $\cos^{-1}(12/13)$

E. $\tan^{-1}(12/5)$