1. If $f(4x-1)=5x+3$, then $f(7)=?$

$f(7)=f(4(2)-1)=5(2)+3=13$

Correct Answer: 13

2. If $\sqrt{x-3}+(y+2)^2+|\ln(z-3)|=0$, find the value of sum $x+y+z$.

$\sqrt{x-3}+(y+2)^2+|\ln(z-3)|=0$ implies $x-3=0$, $x=3$; $y+2=0$, $y=-2$; and $\ln(z-3)=0$, $z-3=1$, $z=4$. $x+y+z=3+(-2)+4=5$

Correct Answer: 5

3. Solve the inequality $\log_{1/2} x>2$. Write your solution in interval notation.

$y=\log_{1/2} x$ is a decreasing function and its domain is $(0,\infty)$. $0<x<\left(\frac{1}{2}\right)^2=\frac{1}{4}$

Correct Answer: $\left(0,\ \frac{1}{4}\right)$

4. For what value(s) of *x* does $x=1+|1-x|$?

When $x<1$, $x=1+1-x$, $2x=2$ and $x=1\geq 1$. No solution.
When $x\geq 1$, $x=1-(1-x)$ and $x=x$ is an identity. The solution set is $[1,\infty)$.

Correct Answer: $x\geq 1$ or $[1,\infty)$

5. Find the decimal number with the property that moving the decimal point one digit to the left and adding 3 yields a number that is half its original value.

Let *x* be the decimal number. Its value becomes $\frac{1}{10}x$ after its decimal point has been moved one digit to the left. We have $3+\frac{1}{10}x=\frac{1}{2}x$, $\frac{2}{5}x=3$, $2x=15$, and $x=7.5$

Correct Answer: 7.5

6. Evaluate the determinant $\begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \end{vmatrix}$

$$\begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \end{vmatrix} = -1 \cdot \begin{vmatrix} 0 & 2 & 0 \\ 3 & 0 & 1 \\ 0 & 0 & 4 \end{vmatrix} = -1 \cdot (-2) \cdot \begin{vmatrix} 3 & 1 \\ 0 & 4 \end{vmatrix} = 24$$

Correct Answer: 24

7. If $P(A) = P(B) = \frac{1}{3}$ and $P(A \mid B) = \frac{1}{6}$, determine the value of $P(A' \cap B')$.

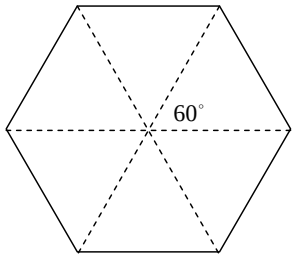
$$P(A \cap B) = P(A \mid B) \cdot P(B) = \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18}$$

$$P(A' \cap B') = 1 - P(A) - P(B) + P(A \cap B) = 1 - \frac{1}{3} - \frac{1}{3} + \frac{1}{18} = \frac{7}{18}$$

Correct Answer: $\frac{7}{18}$

8. Find the area of a regular hexagon with sides of length 5.

$$A = 6\left(\frac{1}{2} \cdot 5^2 \sin 60^\circ\right) = \frac{75\sqrt{3}}{2}$$



Correct Answer: $\frac{75\sqrt{3}}{2}$

9. Given $f(x) = x^2$, $h(x) = f[1 + g(x)]$, $g'(1) = 1$, *and* $h'(1) = 1$ for all real *x*. Determine the numerical value of $g(1)$.

$h(x) = (1 + g(x))^2$, $h'(x) = 2(1 + g(x))g'(x)$, $h'(1) = 2(1 + g(1))g'(1)$, $1 = 2(1 + g(1)) \cdot 1$, $1 + g(1) = \frac{1}{2}$, and $g(1) = -\frac{1}{2}$.

Correct Answer: $-\frac{1}{2}$

10. Who am I?

(1646 – 1716)

Hint: I developed the infinitesimal calculus independently of Isaac Newton.

Correct Answer: Gottfried Leibniz

11. The sum of the first three terms of a positive geometric sequence is equal to seven times the first term. The sum of the first four terms is 45. What is the first term of the sequence?

$S_3 = a_1 \frac{1-r^3}{1-r} = 7a_1$, $r^2 + r + 1 = 7$, $r^2 + r - 6 = 0$, $(r+3)(r-2) = 0$. Since all terms are positive, $r = 2$. $S_4 = a_1 \frac{1-2^4}{1-2} = 45$, $15a_1 = 45$, and $a_1 = 3$.

Correct Answer: 3

12. Find the sum of the real roots of $|x-5|^2 + 2|x-5| = 3$

$|x-5|^2 + 2|x-5| - 3 = 0$, $(|x-5|+3)(|x-5|-2) = 0$, $|x-5| = -3$ (no solution) or $|x-5| = 2$ ($x = 7$ or $x = 3$). $7 + 3 = 10$

Correct Answer: 10

13. Using base 5 notation, determine the following sum: $423_5 + 304_5 + 241_5 + 342_5$

$423_5 + 304_5 + 241_5 + 342_5 = 1232_5 + 241_5 + 342_5 = 2023_5 + 342_5 = 2420_5$

Correct Answer: 2420_5

14. Find the sum of the solutions of the equation: $\log_2 x^{\log_2 x} = 4$

$\log_2 x \cdot \log_2 x = 4,\ (\log_2 x)^2 = 4,$
$\log_2 x = 2,\ x = 2^2 = 4,$
or $\log_2 x = -2,\ x = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$. Then, $4 + \frac{1}{4} = \frac{17}{4}$.

Correct Answer: $\frac{17}{4}$

15. If $\tan\theta = \frac{2}{3}$, then $\cos 2\theta = ?$

$\tan\theta = \frac{2}{3}$ implies $\cos\theta = \pm\frac{3}{\sqrt{2^2+3^2}} = \pm\frac{3}{\sqrt{13}}$. $\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\pm\frac{3}{\sqrt{13}}\right)^2 - 1 = \frac{18}{13} - 1 = \frac{5}{13}$

Correct Answer: $\frac{5}{13}$

16. Twice the complement of an angle is 12° less than one-half the supplement of the angle. What is the measure of the angle?

Let θ be the angle. $2(90^\circ - \theta) = \frac{1}{2}(180^\circ - \theta) - 12^\circ$, $180^\circ - 2\theta = 90^\circ - \frac{1}{2}\theta - 12^\circ$, $102^\circ = \frac{3}{2}\theta$, and
$\theta = \frac{2}{3}\cdot 102^\circ = 68^\circ$.

Correct Answer: 68°

17. A square is positioned on a standard *xy*-rectangular coordinate system such that one vertex is at the origin and the diagonals of the square intersect at the point (2, 3). Find the area of this square.

The length of the diagonal of the square is $2\sqrt{(2-0)^2 + (3-0)^2} = 2\sqrt{13}$
The length of sides of the square is $2\sqrt{13}\cdot\sin 45^\circ = 2\sqrt{13}\cdot\frac{\sqrt{2}}{2} = \sqrt{26}$
The area of the square is 26.

Correct Answer: 26

18. Find the distance between the following two points given in polar coordinates: $(5, 20^\circ)$ and $(3, 140^\circ)$

$$d = \sqrt{(5\cos 20^\circ - 3\cos 140^\circ)^2 + (5\sin 20^\circ - 3\sin 140^\circ)^2}$$
$$= \sqrt{25\cos^2 20^\circ + 9\cos^2 140^\circ + 25\sin^2 20^\circ + 9\sin^2 140^\circ - 30(\cos 20^\circ \cos 140^\circ + \sin 20^\circ \sin 140^\circ)}$$
$$= \sqrt{25 + 9 - 30\cos(140^\circ - 20^\circ)} = \sqrt{34 - (-15)} = \sqrt{49} = 7$$

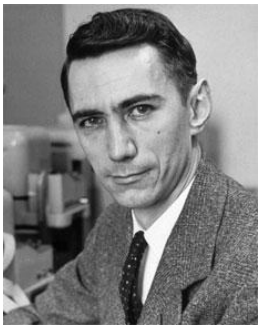
Correct Answer: 7

19. Given $P(A) = \frac{5}{12}$, $P(B) = \frac{1}{6}$, and $P(A \mid B) + P(B \mid A) = \frac{7}{10}$, determine $P(A \cap B)$.

$$\frac{7}{10} = P(A \mid B) + P(B \mid A) = \frac{P(A \cap B)}{P(B)} + \frac{P(A \cap B)}{P(A)} = 6P(A \cap B) + \frac{12}{5}P(A \cap B) = \frac{42}{5}P(A \cap B)$$
$$P(A \cap B) = \frac{7}{10} \cdot \frac{5}{42} = \frac{1}{12}$$

Correct Answer: $\frac{1}{12}$

20. Who am I?



(1916–2001)

Hint: I am known as *the father of information theory*.

Correct Answer: Claude Shannon

(Bonus) Who am I?

Hint: Goddess of Math Bowl for 25 years

Correct Answer: Julia Polk