1. For any positive integer n, $n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$. Determine the value of n if $n! = 30(n-2)!$

   A. 7

   B. 5

   C. 10

   D. 6

   E. 9

2. The picture below shows a regular polygon in gray that is partially covered by a white sheet of paper. How many sides does the polygon have?

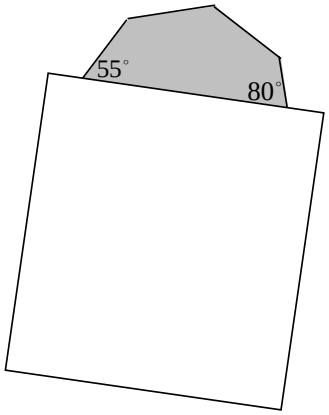
   A. 6

   B. 8

   C. 10

   D. 12

   E. The answer cannot be determined from the given information



3. If the graph of $2x + y + 3 = 0$ is perpendicular to the graph of $3x + ky + 2 = 0$, then k equals what?

   A. -6

   B. $2/3$

   C. 6

   D. $-2/3$

   E. $-3/2$

4. $\frac{4-8x}{x^4+4x^2}$ is identical to which of the following expressions?

A. $\frac{2x}{x^2+4}-\frac{3}{x^2+4}-\frac{2}{x}+\frac{1}{x^2}$

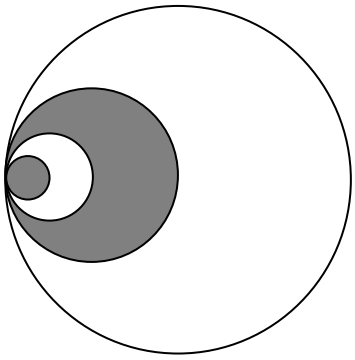
B. $\frac{x}{x^2+4}-\frac{1}{x^2+4}-\frac{3}{x}+\frac{1}{x^2}$

C. $\frac{2x}{x^2+4}-\frac{1}{x^2+4}-\frac{2}{x}+\frac{1}{x^2}$

D. $\frac{2x}{x^2+4}-\frac{1}{x^2+4}-\frac{2}{x}-\frac{3}{x^2}$

E. $\frac{x}{x^2+4}-\frac{1}{x^2+4}-\frac{2}{x}-\frac{1}{x^2}$

5. In the diagram below, the radius of each circle is ½ the radius of the next largest circle. If a point inside the largest circle is chosen at random, what is the probability that it would lie in the shaded region?



A. 1/2

B. 3/16

C. 7/16

D. 13/64

E. 27/64

6. Find the minimum value of the following function of x and y:
$f(x,y)=x^2-6xy+10y^2-10y+40$

A. 40

B. 15

C. 10

D. 5

E. -5

7. If x and y are the smallest possible positive angles for which $\sin x = \frac{1}{\sqrt{5}}$ and $\sin y = \frac{1}{\sqrt{10}}$, then find the value of $(x+y)$.

A. $\pi/6$

B. $\pi/5$

C. $\pi/4$

D. $\pi/3$

E. $\pi/2$

8. Evaluate $\frac{1}{\log_4 6} + \frac{1}{\log_9 6}$

A. 2

B. 4

C. 6

D. 9

E. None of the above

9. (Tie Break No. 1) In trapezoid $ABCD$, $AB \parallel CD$, $AB = 3$, $CD = 6$, and $AD = BC = 5$. Find the length of the diagonal AC.

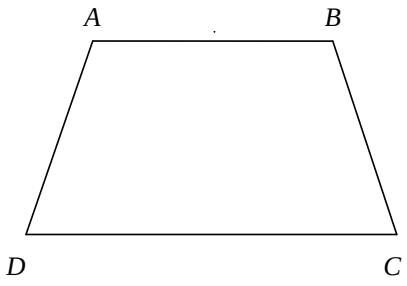
A. $3\sqrt{5}$

B. $\sqrt{43}$

C. $5\sqrt{2}$

D. 7

E. None of the above



10. If $\frac{2^{2011}+2^{2008}}{2^{2010}-2^{2009}}$ is written in the form $\frac{a}{b}$ where a and b are integers with no common divisors, what is $a+b$?

A. 11

B. 2

C. 12

D. 14

E. 38

11. The product of two positive integers is 25 times their quotient. What can you say for sure about this situation?

A. The sum of the numbers is at least 10.

B. The difference of the numbers is at most 10.

C. Nothing can be said for sure.

D. One of the numbers is 5.

E. None of the above.

12. Evaluate: $\left[2\left(\cos\left(\frac{5\pi}{18}\right)+i\sin\left(\frac{5\pi}{18}\right)\right)\right]^6$

A. $32+32i\sqrt{3}$

B. $32-32i\sqrt{3}$

C. $32\sqrt{3}+32i$

D. $32\sqrt{3}-32i$

E. $16\sqrt{3}+16i$

13. What is the distance between the points of intersection of the graphs of $y = x^2 - 4x - 3$ and $x = y + 3$?

A. $\sqrt{5}$

B. $5\sqrt{2}$

C. $\sqrt{10}$

D. $\sqrt{15}$

E. The graphs only intersect at one point.

14. If $\cos x = \frac{3}{5}$ and $\cot x < 0$, find the value of $\frac{\sin x - \tan x}{1 + \sec x}$.

A. $\frac{4}{5}$

B. $-\frac{4}{5}$

C. $-\frac{1}{5}$

D. $\frac{1}{5}$

E. None of the above

15. In ΔABC, D, E, and F are mid-points of AB, BC, and CA, respectively. Find the ratio of the area of ΔDEF to the area of ΔABC.

A. $1:4$

B. $1:2$

C. $1:3$

D. $2:3$

E. None of the above

16. A cow is tied with a 30 foot rope to an outside corner of a rectangular barn having dimensions 20 feet by 10 feet. How many square feet of grazing area does the cow have?

A. 800π

B. 200

C. 900π

D. 675π

E. 775π

17. In ΔABC, $AB = AC$ and CD is the bisector of $\angle C$. If $\angle BDC = 75^\circ$, find $\angle A$.

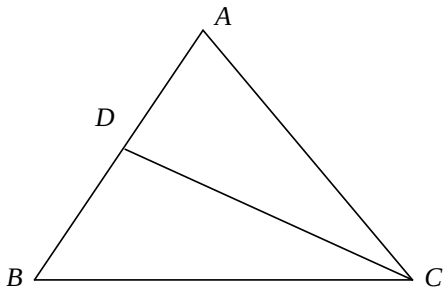
A. 40°

B. 50°

C. 35°

D. 60°

E. 55°



18. (Tie Break No. 2) Given $f(0) = 3; \quad f(n+1) = 2f(n) + 3$. What is $f(10)$?

A. 6771

B. 6241

C. 7142

D. 5763

E. 6141

19. Find the area of an equilateral triangle that is inscribed in a circle if the circumference of the circle is 6π.

A. $\frac{8\sqrt{3}}{3}$

B. $\frac{27\sqrt{3}}{4}$

C. $\frac{3\sqrt{2}}{2}$

D. $\frac{30\sqrt{5}}{7}$

E. $\frac{5\sqrt{3}}{4}$

20. Suppose x is a complex number satisfying the equation $x+\frac{1}{x}=1$. What is the value of $x^3+\frac{1}{x^3}$?

A. -2

B. -1

C. 0

D. 1

E. 2

21. The larger angles of a rhombus are double the size of the smaller angles. The shorter diagonal measures 10 inches. Find the length of a side of this rhombus.

A. 5

B. 7

C. 8

D. 10

E. 12

22. Find the maximum value of the function $f(x) = \cos\left(x + \frac{\pi}{3}\right) + \cos x$.

A. 1

B. $\sqrt{2}$

C. $1/2$

D. 2

E. $\sqrt{3}$

23. A postal employee delivered mail daily for 42 days, each day delivering 4 more letters than on the previous day. The total delivery for the first 24 days of the period was the same as that for the last 18 days. How many letters did the employee deliver during the whole 42-day period?

A. 1000

B. 11120

C. 12096

D. 13028

E. 21434

24. Find the maximum value of the function
$f(x) = x^2 - (x-2)^2 - (x-1)^2 - (x-5)^2 - (x-4)^2 + (x-6)^2$.

A. 3

B. 8

C. 0

D. 5

E. 20

25. If $x = 3\sin\theta$ and $0^\circ \le \theta \le 90^\circ$, write the expression $\dfrac{\sin 2\theta}{4}$ in terms of just x.

A. $\dfrac{x\sqrt{9-x^2}}{18}$

B. $\dfrac{x\sqrt{9-x^2}}{9}$

C. $\dfrac{x}{12}$

D. $\dfrac{x}{6}$

E. $\dfrac{2x\sqrt{9-x^2}}{9}$

26. If the distance from the point (m^2, m) in Quadrant I to the line $y = x - 2$ is $2\sqrt{2}$. What is the value of m?

A. 3

B. 2

C. 1

D. -1

E. -2

27. The diagonals of a quadrilateral are 10 and 8. Find the perimeter of a new quadrilateral formed by joining the midpoints of the sides of the original quadrilateral.

A. 36

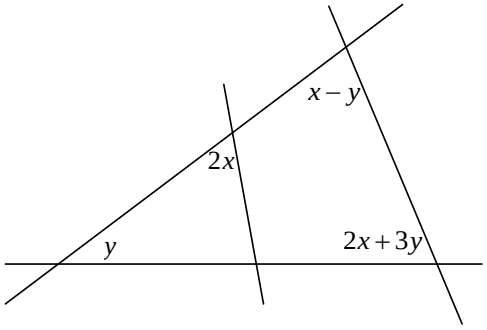
B. 24

C. 20

D. 18

E. 9

28. Given the figure, find the value of $x+y$.



A. 60°

B. 90°

C. 120°

D. 180°

E. 360°

29. An auto insurance company has 10,000 policyholders. Each policyholder is classified as
(i) young or old,
(ii) male or female, and
(iii) married or single.
Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males. How many of the company's policyholders are young, female, and single?

A. 280

B. 423

C. 486

D. 880

E. 896

30. A group of roosters want to buy an alarm clock. If each contributes \$0.35, they lack \$4.40. If each contributes \$0.40, they have \$4.40 extra. The number of roosters is in the range of

A. Less than 50

B. 50 to 100

C. 100 to 150

D. 150 to 200

E. Greater than 200

31. (Tie Break No. 3) Find the value of $\sin 20^\circ \cos 70^\circ + \sin 10^\circ \sin 50^\circ$.

A. $\frac{1}{4}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

E. None of the above

32. Let $f(x) = \frac{x}{\sqrt{1+x^2}}$ and define $f^2(x) = f(f(x))$, $f^3(x) = f(f(f(x)))$, $\cdots\cdots$. Then $f^{99}(1) = ?$

A. $1/4$

B. $1/\sqrt{99}$

C. $1/9$

D. $1/10$

E. None of the above

33. Assume that variables x and y satisfy $|x| + |y| \le 1$. What is the maximum value and the minimum value of $x + 2y$, respectively?

A. 1 and -1

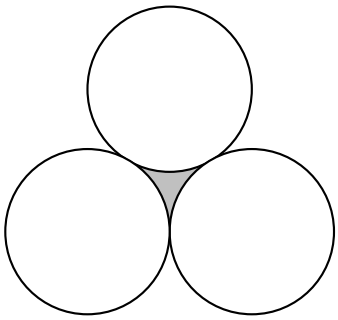
B. $3/2$ and $-3/2$

C. 1 and -2

D. 2 and -2

E. 3 and -3

34. In the figure below, three circles are tangent to each other. Find the area of the shaded region if each circle has a radius of 6.



A. 18

B. $18-4\pi$

C. $36\sqrt{3}$

D. $36\sqrt{3}-18\pi$

E. $36\sqrt{3}-12\pi$

35. If $\tan 2A = 2$ and A terminates in the second quadrant, find the value of $\sin A$.

A. $\dfrac{\sqrt{5}-1}{4}$

B. $\dfrac{\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}$

C. $\dfrac{1+\sqrt{3}}{\sqrt{6+2\sqrt{3}}}$

D. $\dfrac{1+\sqrt{5}}{\sqrt{10+2\sqrt{5}}}$

E. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

36. Find the value of x if $\log_4(\log_{25}(\log_3 x)) = -\dfrac{1}{2}$.

A. $\dfrac{1}{243}$

B. 5

C. 243

D. 25

E. 81

37. From a point five nautical miles due west of a lighthouse, a ship heads due north at a constant speed of 10 knots (nautical miles per hour). How fast in knots is the ship moving away from this lighthouse one hour later?

A. $3\sqrt{5}$

B. $4\sqrt{5}$

C. $2\sqrt{7}$

D. $3\sqrt{7}$

E. 9

38. (Tie Break No. 4) What is the coefficient of $x^3y^2z^3$ in the expansion of $(x-y-2z)^8$?

A. 8

B. -256

C. 560

D. -560

E. -4480

39. If $\cot\varphi = -\frac{4}{3}$ and $\sin\varphi$ is negative, which of the following expressions has the least value?

A. $\cos\left(\frac{\varphi}{2}\right)$

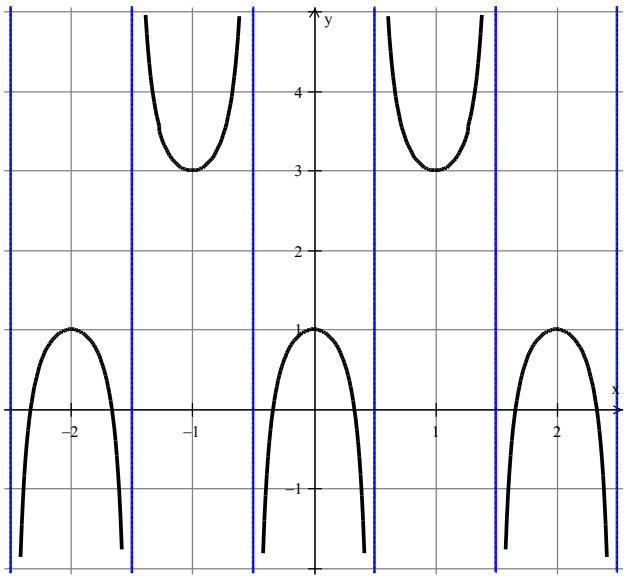
B. $\tan\left(\varphi+\frac{\pi}{4}\right)$

C. $\cos\left(\varphi+\frac{\pi}{6}\right)$

D. $\tan\left(\frac{\varphi}{2}\right)$

E. $\csc\left(\varphi+\frac{3\pi}{2}\right)$

40. Which of the following functions is represented by the graph below?



A. $f(x) = 2 - \sec x$

B. $g(x) = 2 + \sec(\pi x)$

C. $h(x) = 2 - \csc\left(\pi x + \frac{\pi}{2}\right)$

D. $j(x) = 2 - \csc\left(\pi x + \frac{1}{2}\right)$

E. $k(x) = 2 - \csc\left(x + \frac{\pi}{2}\right)$

41. (Tie Break No. 5) The area of triangle $ABC = (BC)^2 + (AC)^2 - (AB)^2$. If angle C is acute, compute the numerical value of its secant.

A. $\sqrt{17}$

B. $2\sqrt{3}$

C. $\sqrt{21}$

D. $3\sqrt{5}$

E. 5

42. Find the solution set of the equation $\log_4|2x+2| - \log_4|3x+1| = \frac{1}{2}$.

A. $\{-1/2, 0\}$

B. $\{0\}$

C. $\{1/2, 0\}$

D. $\{1, -1/2\}$

E. $\{-1/2, 1\}$

43. If $2\tan\left[\cos^{-1}\left(\frac{3}{5}\right)\right] = x$, then x = ?

A. $\frac{3}{4}$

B. $\frac{8}{3}$

C. $\frac{3}{2}$

D. $\frac{3}{8}$

E. $\frac{4}{3}$

44. Find the solution set of the system $\begin{cases} x^2 + y^2 = 2a^2 - 2ab + b^2 \\ 2x^2 - y^2 = a^2 + 2ab - b^2 \end{cases}$ in terms of a and b.

A. $\{(a,\ a-b),\ (-a,\ b-a)\}$

B. $\{(a,\ b-a),\ (-a,\ a-b)\}$

C. $\{(a,\ a-b)\}$

D. $\{(a,\ a-b),\ (a,\ b-a),\ (-a,\ a-b),\ (-a,\ b-a)\}$

E. None of the above

45. Suppose the function $f(x)$ satisfies $f(5) = 3$ and $f(x) = f(x-1) + 3x$. Then $f(3) + f(6) = ?$

A. 0

B. 3

C. -3

D. 6

E. 9