1. Find the sum of all of the digits in the product $2^{2013}\cdot 5^{2011}$.

$2^{2013}\cdot 5^{2011}=2^2(2\cdot 5)^{2011}=4\times 10^{2011}$
The sum of all digits is 4.

Correct Answer: 4

2. Draw two numbers from the set $\{1,2,3,4\}$ at random without replacement. What is the probability of one number being exactly twice the other?

There are two such drawings $\{2,1\}$ and $\{4,2\}$. The Probability is $\frac{2}{{}_4C_2}=\frac{2}{6}=\frac{1}{3}$.

Correct Answer: $\frac{1}{3}$

3. If $x=1+2^p$ and $y=1+2^{-p}$, solve for y in terms of x only.

$x=1+2^p, 2^p=x-1,$ and $2^{-p}=\frac{1}{x-1}$. $y=1+2^{-p}=1+\frac{1}{x-1}=\frac{x}{x-1}$

Correct Answer: $y=1+\frac{1}{x-1}=\frac{x}{x-1}$

4. A certain number is reduced by 7. The result multiplied by 7 is equal to 11 times the result of the number reduced by 11. What is the number?

Let x be the number. $7(x-7)=11(x-11)$, $7x-49=11x-121$, $4x=72$, and $x=18$.

Correct Answer: 18

5. For what values of *k*, does the entire graph of the parabola $y=kx^2-2kx-3$ lie entirely below the *x*-axis? Write your answer in interval notation.

The parabola has to open downward and cannot have real zeros. Therefore, $k<0$ and the discriminant $(-2k)^2-4\cdot k\cdot(-3)<0$. $4k^2+12k<0$, $4k(k+3)<0$, and $-3<k<0$.

Correct Answer: $(-3,0)$

6. Find the real solution of logarithmic equation $\log_3 x + \log_3 (x-6) = 3.$

$\log_3 x(x-6) = 3,\ x(x-6) = 3^3,\ x^2 - 6x - 27 = 0,\ (x-9)(x+3) = 0.\ \log_3(-3)$ is not real number.
$x = 9$

Correct Answer: 9

7. If $(3,-2)$ is the only maximum point of function $y = f(x)$, what is the maximum point of $y = 2f(3x+2) - 1$?

$3x + 2 = 3,\ 3x = 1,\ x = \frac{1}{3};\ y = 2 \cdot (-2) - 1,\ y = -5.$ The maximum point is $\left(\frac{1}{3}, -5\right).$

Correct Answer: $\left(\frac{1}{3}, -5\right)$

8. Consider the piecewise function $f(x) = \begin{cases} x^2 + 1 & x \le 1 \\ \frac{2}{x} & x > 1 \end{cases}.$ Find $f(f(3))$.

$f(3) = \frac{2}{3},\ f(f(3)) = \left(\frac{2}{3}\right)^2 + 1 = \frac{4}{9} + 1 = \frac{13}{9}$

Correct Answer: $\frac{13}{9}$

9. On a test the passing students had an average of 83, while the failing students had an average of 55. If the overall class average was 76, what percent of the class passed?

Let x be the number of the passing students and y the number of failing students.
$83x + 55y = 76(x+y),\ 7x = 21y,\ x = 3y,\ 4x = 3(x+y),$ and $\frac{x}{x+y} = \frac{3}{4} = 75\%$

Correct Answer: 75%

10. Who am I?

1170-1250

Hint: Hare raising experiment

Correct Answer: Leonardo Fibonacci

11. The first two terms of an arithmetic progression are $\log_2 3$ and $\log_2 9$. If the fifth term is *x*, compute the numerical value of 2^x.

Because the second term $\log_2 9 = 2\log_2 3$, the common difference is $\log_2 3$. The fifth term is $x = 5\log_2 3 = \log_2 3^5 = \log_2 243$. $2^x = 2^{\log_2 243} = 243$.

Correct Answer: 243

12. Find the sum of the following in simplest form: $\frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^3}+\frac{2}{7^4}+\frac{1}{7^5}+\frac{2}{7^6}+\cdots$

$$\left(\frac{1}{7}+\frac{1}{7^3}+\frac{1}{7^5}+\cdots\right)+\left(\frac{2}{7^2}+\frac{2}{7^4}+\frac{2}{7^6}+\cdots\right)$$
$$=\frac{1}{7}\cdot\frac{1}{1-\frac{1}{7^2}}+\frac{2}{7^2}\cdot\frac{1}{1-\frac{1}{7^2}}=\frac{1}{7}\cdot\frac{49}{48}+\frac{2}{49}\cdot\frac{49}{48}$$
$$=\frac{7}{48}+\frac{2}{48}=\frac{3}{16}$$

Correct Answer: $\frac{3}{16}$

13. How many terms are in the following sequence: 10, 17, 24, 31, . . . , 374?

$a_1 = 10,\ d = 7,$ and $a_n = 10 + (n-1)\cdot 7 = 7n + 3.$
$7n + 3 = 374,\ 7n = 371,$ and $n = 53$

Correct Answer: 53

14. If $P = 2^{2013} + 2^{-2013}$ and $Q = 2^{2013} - 2^{-2013}$, compute $P^2 - Q^2$.

$P - Q = 2^{2013} + 2^{-2013} - 2^{2013} + 2^{-2013} = 2 \cdot 2^{-2013}$
$P + Q = 2^{2013} + 2^{-2013} + 2^{2013} - 2^{-2013} = 2 \cdot 2^{2013}$
$P^2 - Q^2 = (P - Q)(P + Q) = (2 \cdot 2^{-2013})(2 \cdot 2^{2013}) = 4$

Correct Answer: 4

15. Let $f(x) = \ln\left(x + \sqrt{1 + x^2}\right)$. Determine the value of $f^{-1}(\ln 7)$.

Let $y = f^{-1}(\ln 7)$. Then $\ln\left(y + \sqrt{1 + y^2}\right) = \ln 7$.
$y + \sqrt{1 + y^2} = 7,\ \sqrt{1 + y^2} = 7 - y,\ 1 + y^2 = 49 - 14y + y^2,$
$14y = 48,\ y = \frac{24}{7}.$

Correct Answer: $\frac{24}{7}$

16. Determine the value of the following in the simplest form: $\log_{0.25} 128$

Let $x = \log_{0.25} 128$. $0.25^x = 128,\ \left(\frac{1}{4}\right)^x = 2^7,\ 2^{-2x} = 2^7,$ and $x = -\frac{7}{2}$

Correct Answer: $-\frac{7}{2}$ or -3.5

17. Let *x* and *y* be positive numbers not equal to 1. Then $\dfrac{\log_x y^2}{\log_{x^2} y} = ?$

$$\frac{\log_x y^2}{\log_{x^2} y} = \frac{2\log_x y}{\dfrac{\log_x y}{\log_x x^2}} = 2\log_x y \cdot \frac{\log_x x^2}{\log_x y} = 2 \cdot 2 \cdot 1 = 4$$

Correct Answer: 4

18. For what value of *k* is the polynomial $P(x) = x^{1000} + 3x^9 + kx + 9$ divisible by $(x+1)$?

We have to have $P(-1) = 0$

$P(-1) = (-1)^{1000} + 3(-1)^9 + k(-1) + 9 = 0$

$1 - 3 - k + 9 = 0, \quad k = 7$

Correct Answer: 7

19. What is the sum of the three positive integers *x*, *y*, and *z* that satisfy

$$x + \frac{1}{y + \frac{1}{z}} = 5.4?$$

$x + \dfrac{1}{y + \dfrac{1}{z}} = 5.4$ and $\dfrac{1}{y + \dfrac{1}{z}} < 1$ imply $x = 5$. $\dfrac{1}{y + \dfrac{1}{z}} = 0.4 = \dfrac{1}{2.5}$. $y + \dfrac{1}{z} = 2.5$ implies $y = 2$ and $z = 2$.

$x + y + z = 9$

Correct Answer: 9

20. Who am I?

1707 –1783

Hint: The number *e* is in honor of my name.

Correct Answer: Leonhard Euler