1. Four numbers are written in a row. The average of the first two numbers is 5. The average of the middle two numbers is 4, and the average of the last two numbers is 10. What is the average of the first and last numbers?

A. 9

B. 11

C. 10

D. 10.5

E. 11.5

2. Using the following figure, determine the value of $m\angle A + m\angle B + m\angle C + m\angle D$.

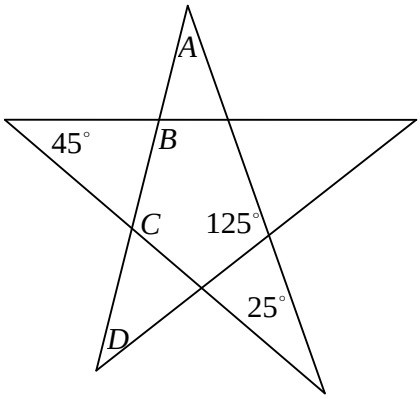
A. 220°

B. 240°

C. 265°

D. 280°

E. 290°



3. Find the value of $x \sin x$ if $x = \frac{\pi}{6}$.

A. $\pi/12$

B. $\pi\sqrt{3}/12$

C. $\pi\sqrt{2}/12$

D. $\pi/6$

E. None of the above

4. The Body Mass Index (BMI) varies directly with a person's weight (pounds) and inversely with the square of the person's height (inches). Given that a 6-foot tall man weighing 180 pounds has a BMI of 24.0, what is the BMI of a woman weighing 120 pounds with a height of 5 feet 4 inches?

A. 20.25

B. 21.50

C. 19.25

D. 23.00

E. 19.85

5. A circle of circumference 18π is inscribed in a regular hexagon as shown below. Find the shaded area.

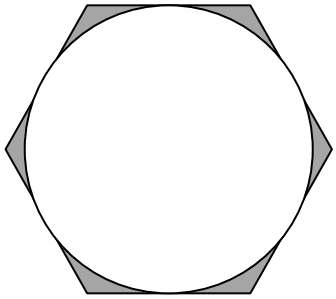
A. $81(3\sqrt{3}-\pi)$

B. $27(8\sqrt{3}-3\pi)$

C. $81(2\sqrt{3}-\pi)$

D. $27(4\sqrt{3}-2\pi)$

E. $9(4\sqrt{3}-2\pi)$



6. (Tie Break No.1) Solve the rational inequality: $\dfrac{5}{x-2} \geq \dfrac{3}{x+2}$

A. $(-\infty,-6]\cup(2,\infty)$

B. $[-6,-2)\cup(2,\infty)$

C. $[-8,-2)\cup(2,\infty)$

D. $(-\infty,-8)\cup(2,\infty)$

E. $(-8,-2)\cup(2,\infty)$

7. Let $\{a_n\}$ be an arithmetic sequence and $\{s_n\}$ be its partial sum sequence. If $\lim_{n\to\infty} \frac{s_n}{n^2} = 4$, find the common difference of $\{a_n\}$.

A. 4

B. 2

C. 6

D. 8

E. Not enough information

8. Find the value of $(\log_2 x)^2$ if $\log_2(\log_8 x) = \log_8(\log_2 x)$.

A. 0

B. 16

C. 27

D. 81

E. $3\sqrt{3}$

9. Solve for *x*, given $\begin{vmatrix} x+a & x & x \\ x & x+b & x \\ x & x & x+c \end{vmatrix} = 0$

A. $x = a+b+c$

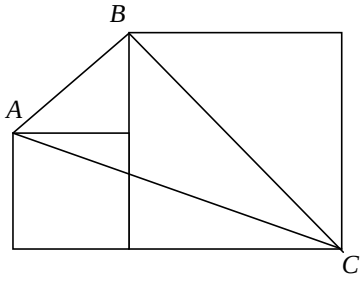
B. $x = \frac{1}{a+b+c}$

C. $x = -\frac{abc}{ab+bc+ca}$

D. $x = \frac{ab+bc+ca}{abc}$

E. $x = \frac{1}{ab+bc+ca}$

10. In the figure, the length of sides of the larger square is *a*. The length of sides of the smaller square is *b*. Find the area of the triangle *ABC*.



A. $\dfrac{ab}{2(a+b)}$

B. $\dfrac{b^2}{2(a+b)}$

C. $\dfrac{a^2}{2(a+b)}$

D. $\dfrac{1}{2}a^2$

E. None of the above

11. Find the solution set of the equation $|3x+2|+|3x-5|=7$

A. $\{-2/3,\ 5/3\}$

B. $[-2/3,\ 5/3]$

C. $(-2/3,\ 5/3)$

D. $(-2/3, 5/3]$

E. None of the above

12. How many subsets does the set of all the rational solutions of the equation $3x^4 - x^3 - 11x^2 + 3x + 6 = 0$ have?

A. 4

B. 2

C. 1

D. 8

E. 16

13. Find the range of the function $f(x)=\frac{1}{2}(\sin x+\cos x)-\frac{1}{2}|\sin x-\cos x|$

A. $[-\sqrt{2}/2, 1]$

B. $[-1, \sqrt{2}/2]$

C. $[-1, 1]$

D. $[-1, \sqrt{3}/2]$

E. $[-\sqrt{3}/2, 1]$

14. (Tie Break No.2) How many positive integer solutions does the equation $x+y+z+w=15$ have?

A. 728

B. 182

C. 324

D. 546

E. 364

15. Find the maximum value of the function $f(x)=\sqrt{x+2}+\sqrt{8-2x}$.

A. $3\sqrt{2}$

B. $3\sqrt{3}$

C. $\sqrt{3}+\sqrt{6}$

D. $\sqrt{5}+\sqrt{2}$

E. None of the above

16. Find the value of $\sin 15^\circ$.

A. $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

B. $\dfrac{\sqrt{2}+\sqrt{6}}{4}$

C. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

D. $\dfrac{\sqrt{6}-\sqrt{2}}{2}$

E. $\dfrac{1}{4}$

17. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} = ?$

A. $1/2$

B. $2/3$

C. $1/4$

D. $1/3$

E. $2/5$

18. Let *a*, *b*, and *c* be positive integers. If $c = (a+bi)^2 - 46i$, where $i^2 = -1$, then $a+b+c = ?$

A. 624

B. 548

C. 552

D. 456

E. 349

19. Solve for the matrix A if $\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} A + \begin{bmatrix} 4 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -2 & 0 \end{bmatrix}$.

A. $\begin{bmatrix} 15 & -11 \\ -2 & 1 \end{bmatrix}$

B. $\begin{bmatrix} -2 & -11 \\ 1 & 15 \end{bmatrix}$

C. $\begin{bmatrix} -2 & 15 \\ 1 & -11 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 15 \\ -2 & -11 \end{bmatrix}$

E. None of the above

20. Find the sum $8 + 5 + 2 - 1 - \cdots\cdots - 340$.

A. -15624

B. -21232

C. -19426

D. -19422

E. -18422

21. (Tie Break No.3) Find the area of the region enclosed by $y = 0$ and a quadratic function $y = f(x)$ that has vertex $(3, 2)$ and passes through $(1, 0)$.

A. $16/3$

B. 5

C. $15/4$

D. 6

E. None of the above

22. If $a^2 + b^2 + c^2 + 1 = ab + \frac{3}{2}bc + c$, then $a = ?$

A. $1/2$

B. $1/4$

C. 2

D. 1

E. a can be any real number

23. Among all the pairs of numbers (x, y) such that $2x + y = 20$, find the pair for which the sum of the squares is minimum.

A. $(4, 12)$

B. $(10, 0)$

C. $(7, 6)$

D. $(6, 8)$

E. None of the above

24. If $\sin\theta + \cos\theta = \frac{1}{2}$, what is the value of $\sin^3\theta + \cos^3\theta$?

A. $3/8$

B. $5/16$

C. $1/8$

D. $5/8$

E. $11/16$

25. A man bought some horses for \$900. After training them, he sold all but one to a dude-ranch for \$1,710, thereby realizing a gain of 100% on the horses he sold. How many horses did he buy?

A. 6

B. 8

C. 10

D. 12

E. 14

26. (Tie Break No.4) Find the remaining real zero for the 4^{th} degree polynomial function with the following properties: it only has real coefficients, $1/2$ and $1 + 2i$ are two of its zeros, its *y*-intercept is -30, and its leading coefficient is 2.

A. 8

B. 6

C. -8

D. -6

E. Cannot be determined

27. A water tank is in the shape of an inverted right circular cone, with the height of the cone equal to the diameter of the base. If water is poured into the tank until its height reaches $1/2$ that of the tank, find the ratio of the volume of water to the volume of the tank.

A. $1/2$

B. $2/3$

C. $1/8$

D. $7/8$

E. Not enough information is provided

28. Suppose the measure of an exterior angle of a regular polygon is 24°. How many sides does the polygon have?

A. 12

B. 15

C. 18

D. 20

E. 24

29. The supplement of a certain angle divided by 3 times its complement equals $5/3$. Find the degree measure of the angle.

A. 22.5°

B. 67.5°

C. 112.5°

D. 157.5°

E. None of the above

30. Two chords intersect within a circle. The segments of the first chord are 3 and 11. One segment of the second chord is 4. Find the length of the other segment of the second chord.

A. 12

B. 10

C. 8

D. 6

E. None of the above

31. If a square and a circle have the same perimeter, then the ratio of the area of the square to the area of the circle is

A. $\pi/4$

B. $\pi/2$

C. $\pi^2/2$

D. $\pi^2/4$

E. None of the above

32. Write the trigonometric expression $\sin(\sin^{-1} u - \cos^{-1} v)$ into an algebraic expression in terms of *u* and *v*. You may assume $-1 \le u \le 1 \text{and} -1 \le v \le 1$.

A. $uv + \sqrt{1-u^2}\sqrt{1-v^2}$

B. $uv - \sqrt{1-u^2}\sqrt{1-v^2}$

C. $u\sqrt{1-v^2} + v\sqrt{1-u^2}$

D. $u\sqrt{1-v^2} - v\sqrt{1-u^2}$

E. $u\sqrt{1-u^2} + v\sqrt{1-v^2}$

33. The sum of the lengths of the twelve edges of a closed rectangular box is 140, and the distance from one corner of the box to the farthest corner is 21. The total surface area of the box is

A. 784

B. 776

C. 798

D. 800

E. 812

34. Find the sum of all x-values satisfying the equation $2\cos 3x + 1 = 0$ on the interval $[0, \pi]$.

A. $29\pi/18$

B. $14\pi/9$

C. $29\pi/6$

D. $14\pi/3$

E. $8\pi/3$

35. Two pulleys, one with radius 3 inches and the other with radius 10 inches, are connected by a belt. If the 3-inch pulley is caused to rotate at 3 revolutions per minute, at how many revolutions per minute does the 10-inch pulley rotate?

A. $3/20$ rpm

B. $3/10$ rpm

C. $9/10$ rpm

D. $9/20$ rpm

E. $10/3$ rpm

36. $\tan^{-1} 1 + \tan^{-1}(1/2) = ?$

A. $\sin^{-1}\left(1/\sqrt{10}\right)$

B. $\cos^{-1}\left(1/\sqrt{10}\right)$

C. $\cos^{-1}\left(3/\sqrt{10}\right)$

D. $\tan^{-1}(3/2)$

E. $\sin^{-1}\left(1/\sqrt{5}\right)$

37. (Tie Break No.5) Suppose $\sin x = \cos y$, $\sqrt{6}\sin y = \tan z$, and $2\sin z = \sqrt{3}\cos x$. Let *u*, *v*, and *w* denote $\sin^2 x$, $\sin^2 y$, and $\sin^2 z$ respectively. Find the value of the ordered triple (u, v, w).

A. $(1/2, 1/2, 1/2)$

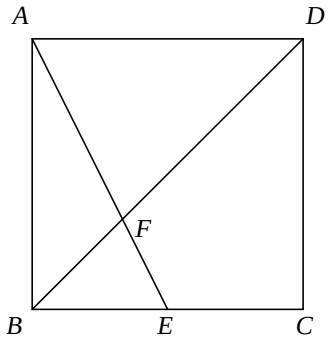
B. $(3/4, 1/4, 1/2)$

C. $(1/2, 1/2, 3/4)$

D. $(1/4, 3/4, 1/2)$

E. $(1, 0, 0)$

38. In the figure, *ABCD* is a square with side length *a*. *E* is the midpoint of *BC*. *AE* intersects the diagonal *BD* at *F*. Find the area of triangle *ABF*.



A. $a^2/5$

B. $a^2/8$

C. $a^2/6$

D. $2a^2/9$

E. None of the above

39. Solve the compound inequality: $3x + 3 > 4x$ or $2(x + 3) > 6$.

A. $(0, 3)$

B. $(-\infty, \infty)$

C. $(-\infty, 0) \cup (3, \infty)$

D. $(0, 3) \cup (3, \infty)$

E. ϕ

40. Let $f(x) = \cos^2 x,\ g(x) = \frac{1}{x},$ and $h(x) = \sin x$. What is $\left(f \circ \left(g^{-1} \circ h\right)^{-1}\right)(2)$?

A. $1/4$

B. $3/4$

C. 1

D. $\sqrt{3}/2$

E. 3

41. Let both x and y be two-digit, positive integers whose mean is 80. What is the minimum possible value of their product xy ?

A. 1600

B. 6000

C. 6241

D. 6039

E. 6400

42. What is the value of the following series: $\sum_{n=0}^{\infty} \frac{(-1)^n n}{1-4n^2}$?

A. 0

B. $1/4$

C. $-1/4$

D. $-1/3$

E. None of the above, it diverges.

43. Let $x=\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}$ and $y=\sqrt{6-\sqrt{6-\sqrt{6-\cdots}}}$. Find $x+y$

A. 6

B. 0

C. $2\sqrt{6}$

D. $2\sqrt{3}$

E. 5

44. If $3\sin\theta+4\cos\theta=5$, then $\tan\theta=?$

A. 1

B. -1

C. $3/4$

D. $4/3$

E. 0

45. Given an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(0<b<a)$, V_1 and V_2 are its left vertex and right vertex respectively. F_1 and F_2 are its left focus and right focus respectively. Let $|V_1F_1|$, $|F_1F_2|$, and $|F_1V_2|$ denote the distances between these points. They form a geometric sequence. Find the eccentricity of the ellipse.

A. $1/\sqrt{5}$

B. $1/4$

C. $1/2$

D. $3/5$

E. $\sqrt{5}$