1. There are several dogs and people in a dog park. A cat goes by, is barked at, and notices that there are 82 feet and 26 heads (the cat is not included). How many dogs are there?

   A. 10

   B. 12

   C. 8

   D. 18

   E. 15 <<<<<

2. At the halfway point of a road trip, it is observed that the average speed so far has been 50 miles per hour. What would the average speed for the remainder of the trip need to be so that the average speed for the *entire* trip would be 60 miles per hour?

   A. 68

   B. 70.

   C. 72

   D. 80

   E. None of the above <<<<

3. **(Tie Break No 1)** In the figure, $AD \parallel BC$, $AB = CD$, $AC \perp CD$, $BC = 7$, and $\angle D = 60^\circ$. Find the length of AD.

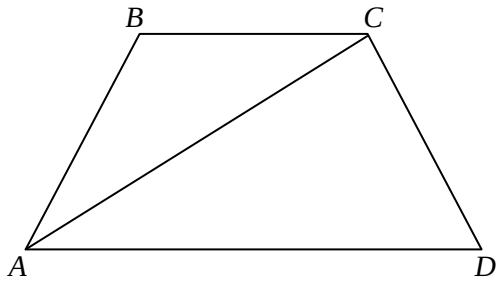
   A. $7\sqrt{3}$

   B. 14 <<<<

   C. 15

   D. $14\sqrt{2}$

   E. None of the above



4. Let $f(x, y)$ be defined by (i) $f(x, 0) = x$ and (ii) $f(x, y+1) = f(f(x, y), y)$. Which of the following is the largest?

   A. $f(14,11)$ <<<<<

   B. $f(10,15)$

   C. $f(11,14)$

   D. $f(13,12)$

   E. $f(12,13)$

5. In a squadron of 150 airmen, 60 have had chicken pox, 50 have had measles and 40 have had mumps. Also, 30 have had chicken pox and measles, 15 have had chicken pox and mumps while 10 have had measles and mumps. If 5 airmen have had all three diseases, how many airmen have had none of the three?

   A. 10

   B. 35

   C. 0

   D. 15

   E. 50 <<<<<

6. Two trains face each other on the same track, 100 miles apart. Train A starts moving toward B at 10 mph while train B starts moving toward A at 30 mph. Mighty Mouse starts on Train A's headlight and flies toward B at a rate of 70 mph. After he hits B, he turns around and flies back to A, again at 70 mph. When he reaches A what is the total number of miles that Mighty Mouse has traveled?

   A. 70

   B. 100

   C. 87.5

   D. 122.5 <<<<<

   E. 140

7. Evaluate $\cos\left(\sin^{-1}\frac{3}{5}+\frac{\pi}{6}\right)$

A. $\frac{4\sqrt{3}-3}{10}$ <<<<<

B. $\frac{4-3\sqrt{3}}{10}$

C. $\frac{3\sqrt{3}-4}{10}$

D. $\frac{3-4\sqrt{3}}{10}$

E. $\frac{3-4\sqrt{3}}{7}$

8. A stock broker trades stocks over 100 days. On day 1 he loses \$100. Every day after the first, he loses \$100 more than on the previous day. What is the average daily loss during the 100 day period?

A. \$10,000

B. \$100

C. \$5050 <<<<<

D. \$500

E. \$1100

9. If A and B are acute angles, $\cos(A+B)=\frac{4}{5}$, and $\sin(A-B)=\frac{12}{13}$, find the value of $\cos 2A$.

A. 63/65

B. – 63/65

C. 33/65

D. 56/65

E. –16/65 <<<<

10. **(Tie Break No 2)** If *k* is a positive real number and the coefficient of x^4 term in the expansion of $\left(x^2+\frac{k}{x}\right)^5$ is 50, then the value of *k* is

A. 2

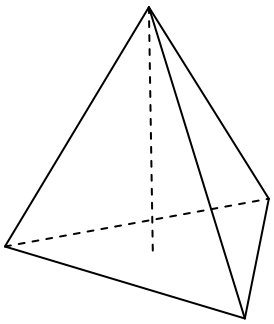
B. $\sqrt{2}$

C. $1/2$

D. 5

E. $\sqrt{5}$ <<<<

11. A tetrahedron is made of an equilateral triangle base and three congruent isosceles triangles. If the length of sides of its base is 2 and its height is 1, find the surface area of the tetrahedron.



A. $3\sqrt{3}$ <<<<

B. 5

C. $2\sqrt{3}$

D. $3+\sqrt{3}$

E. None of the above

12. Find the solution set of the equation $\sqrt{\left(\frac{x}{2x-1}\right)^2}=\frac{x}{1-2x}$.

A. $(0,1/2)$

B. $\{0\}$

C. $(-\infty,0]\cup(1/2,\infty)$

D. $[0,1/2)$ <<<<

E. None of the above

13. Let $\{a_n\}$ be an arithmetic sequence with $a_1 = 25$ and non-zero common difference. If a_1, a_{11}, and a_{13} form three consecutive terms in a geometric sequence, find the sum of first $3n$ terms of the arithmetic sequence.

A. $S_{3n} = 78n - 9n^2$ <<<<

B. $S_{3n} = 81n - 9n^2$

C. $S_{3n} = 75n - 9n^2$.

D. $S_{3n} = 72n - 9n^2$

E. None of the above

14. Let $A = \{x \mid x^3 - 2x^2 + 4x - 8 < 0\}$, $B = \{x \mid x^2 - 5x - 6 \geq 0\}$, and M^C denoted the complementary set of M in the real number set $\boldsymbol{R}$. Find $A \cup B^C$

A. $(-\infty, 2)$

B. $(-\infty, -2) \cup [-1, 6]$

C. $[-1, 2)$

D. $(-\infty, 6]$

E. $(-\infty, 6)$ <<<<

15. The area of square $ABCD$ is 90. The point P is on AB. $PB = 2AP$. The points M and N are on the diagonal BD. If $BM == \frac{1}{2} MN = ND$, find the area of ΔPMN

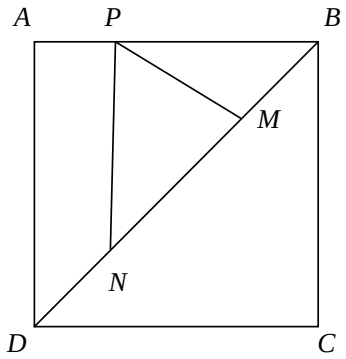
A. 18

B. 15 <<<<

C. 12

D. $9\sqrt{2}$

E. $6\sqrt{2}$



16. If $a_{n+1} = \dfrac{n}{a_n}$ for $n \geq 1$ and $a_1 = 57$, find the product $a_1 a_2 \cdots a_{10}$

A. 570

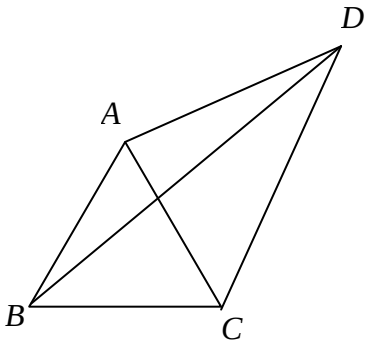
B. 945 <<<<

C. 10395

D. 627

E. 2835

17. In the figure, the ΔABC is an equilateral triangle. $\angle ADC = 30^\circ$, $AD = 3$, and $BD = 5$. Find the length of CD.



A. $3\sqrt{2}$

B. 4 <<<<

C. $2\sqrt{5}$

D. 4.5

E. None of the above

18. Find the x-coordinate of the point where the graph of $f(x) = \dfrac{x^3 - 3x^2 + 5x + 1}{x^2 + 3x + 4}$ intersects its oblique asymptote.

A. $-4/11$

B. $-9/13$

C. $-16/17$

D. $-25/19$ <<<<

E. Graphs of functions do not intersect their asymptotes.

19. If an annual interest rate *r* will increase the value of an investment by 60% in *t* years when interest is compounded continuously, then by what percentage will an interest rate $2r$ increase the value of an investment in *2t* years if interest is compounded continuously?

A. 111.46%

B. 222.36%

C. 333.76%

D. 444.26%

E. 555.36% <<<<

20. Let f be the real function defined by $f(x) = x^2 - 8x + 19$ and g the real function defined by the set $g = \{(1, 2), (3, 4), (5, 6), (7, 8), (9, 10)\}$. How many elements are in the domain of the real composition function $g \circ f$?

A. 5

B. 6

C. 7 <<<<

D. 8

E. ∞

21. Euclid asks his friends to guess the value of a positive integer *n* that he has chosen. Archimedes guesses that *n* is a multiple of 10. Euler guesses that *n* is a multiple of 12. Fermat guesses that *n* is a multiple of 15. Gauss guesses that *n* is a multiple of 18. Hilbert guesses that *n* is a multiple of 30. Exactly two of the guesses are correct. Which persons have guessed correctly?

A. Archimedes and Hilbert

B. Gauss and Hilbert

C. Euler and Gauss <<<<<

D. Fermat and Gauss

E. Archimedes and Fermat

22. Find the distance between the line $2x+y=-2$ and the center of the circle $x^2+y^2-8x-10y+37=0$.

A. $2\sqrt{11}$

B. $3\sqrt{5}$ <<<<

C. $4\sqrt{3}$

D. $5\sqrt{2}$

E. None of the above

23. In the figure, ΔABC is a right triangle with $C=90^\circ$ and $AC=BC=2\sqrt{2}$. If D is a point on AB and $DC=\sqrt{5}$, which of the following could be the length of DB?

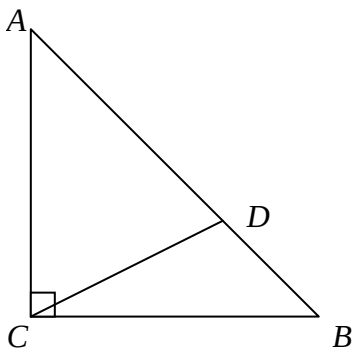
A. $\frac{7}{4}$

B. $\frac{5}{3}$

C. 3 <<<<

D. $\frac{3}{2}$

E. None of the above



24. The parabolas $y=x^2$ and $y=2x^2-3x+2$ intersect at two points. What is the slope of the straight line through the two points of intersection?

A. -2

B. 1

C. 0

D. 4

E. 3 <<<<<

25. **(Tie Break No 3)** Which of following equalities is true for $x \in [-1, 0]$.

A. $\pi - \cos^{-1}(-x) = \sin^{-1}\sqrt{1-x^2}$

B. $\pi - \sin^{-1}(-x) = \cos^{-1}\sqrt{1-x^2}$

C. $\pi - \cos^{-1} x = \sin^{-1}\sqrt{1-x^2}$ <<<<

D. $\pi - \sin^{-1} x = \cos^{-1}\sqrt{1-x^2}$

E. None of the above

26. Given $\cos x \cos y + \sin x \sin y = \frac{1}{2}$ and $\sin 2x + \sin 2y = \frac{2}{3}$, find $\sin(x+y)$.

A. $3/5$

B. $4/5$

C. $2/3$ <<<<

D. $\left(2+\sqrt{3}\right)/5$

E. $-2/3$

27. If the absolute value of complex number *z* is 2, what is the maximum value of the absolute value of $(z-i)$?

A. 1

B. 2

C. $\sqrt{5}$

D. 3 <<<<

E. $\sqrt{8}$

28. Which of the following functions is decreasing on $(-\infty, 0)$?

A. $f(x) = -\frac{x^3}{3} + x$

B. $f(x) = \log_{1/2}(-x)$

C. $f(x) = \left(\frac{1}{2}\right)^{-x}$

D. $f(x) = (x+1)^2$

E. $f(x) = \frac{x}{x-1}$ <<<<<

29. If $A = 20^\circ$ and $B = 25^\circ$, then $(1+\tan A)(1+\tan B) = ?$

A. $\sqrt{3}$

B. 2 <<<<<<<<<<

C. $1+\sqrt{2}$

D. 4

E. none of the above

30. Find all values of x such that $\tan\left(\frac{1}{3}x - \frac{\pi}{2}\right) = \frac{1}{\sqrt{3}}$, where k is any integer.

A. $x = \left(\frac{3k-1}{2}\right)\pi$

B. $x = (6k+2)\pi$

C. $x = (3k+1)\pi$

D. $x = \left(\frac{6k-1}{2}\right)\pi$

E. $x = (3k+2)\pi$ <<<<<<

31. **(Tie Break No 4)** Evaluate $\dfrac{\cos 2^\circ + \cos 32^\circ + \cos 58^\circ + \cos 88^\circ}{\cos 28^\circ}$

A. 0

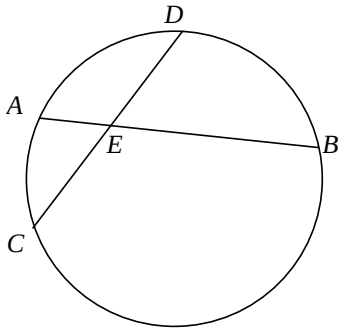
B. $\sqrt{3}-1$

C. 1

D. $\sqrt{3}+1$ <<<<<

E. 2

32. In the figure, AB and CD are chords of the circle, and the chords intersect each other at point E. If $AE=8$, $EB=6$, $DC=16$, and $DE>EC$, find the value of $DE-EC$.



A. 16

B. 8 <<<<

C. 2

D. $2\sqrt{3}$

E. 3

33. If the domain of $f(x)=\log_2\left(x^2-kx+k\right)$ is $(-\infty,\infty)$ and $f(x)$ is decreasing on $(-\infty,1)$, find the range of real parameter k.

A. $0<k<4$

B. $2\le k<4$ <<<<

C. $2\le k\le 4$

D. $2\le k\le 3$

E. $1<k<5$

34. A circle of circumference 3π is inscribed in a regular hexagon. Find the area of the hexagon.

A. $18\pi^2$

B. $\frac{9\pi\sqrt{3}}{2}$

C. $9\sqrt{2}$

D. $\frac{9\sqrt{3}}{2}$ <<<<<<

E. 18π

35. Find the measure of the central angle of a regular polygon if the sum of the measures of its interior angles is 2700°

A. 15°

B. 17°

C. 24°

D. $17\frac{1}{2}^\circ$

E. $21\frac{3}{17}^\circ$ <<<<

36. In ΔABC, $b = 2$, $B = \frac{\pi}{6}$, and $C = \frac{\pi}{4}$. Find the area of the triangle

A. $2\sqrt{3} + 2$

B. $\sqrt{3} + 1$ <<<<

C. $\sqrt{3} - 1$

D. $2\sqrt{3} - 2$

E. None of the above

37. Given $\sin 2A = \frac{2}{3}$, find the value of $\cos^2\left(A + \frac{\pi}{4}\right)$

A. $1/6$ <<<<

B. $1/3$

C. $1/2$

D. $2/3$

E. $5/6$

38. Let $f(x)$ be an even function and increasing on the interval $[0, \pi]$. Which of the following inequalities is true?

A. $f(\log(1/100)) > f(-\pi/2) > f(-2\pi/3)$

B. $f(-2\pi/3) > f(-\pi/2) > f(\log((1/100))$

C. $f(-2\pi/3) > f(\log(1/100)) > f(-\pi/2)$ <<<<

D. $f(-\pi/2) > f(\log(1/100)) > f(-3\pi/2)$

E. None of the above

39. In which of the following intervals, we have $\sin^{-1} x > \cos^{-1} x$?

A. $\left[0, \frac{\sqrt{2}}{2}\right)$

B. $\left(\frac{\sqrt{2}}{2}, 1\right]$ <<<<

C. $\left(-1, \frac{\sqrt{2}}{2}\right]$

D. $[-1, 0)$

E. None of the above

40. You are 5 feet 6 inches tall. In the middle of the afternoon your shadow is 4 feet long. Your pet giraffe's shadow is 11.2 feet long. How tall is your giraffe?

A. 15.4 feet <<<<<

B. 22.4 feet

C. 8 feet

D. 14.3 feet

E. 19.1 feet

41. Given $f(x) = x^3 + ax^2 + bx + c$, which of the following statements is **FALSE**?

A. There is a real number x_0 such that $f(x_0) = 0$

B. If $f(x)$ has relative maximum point at x_0, then $f(x)$ is increasing on $(-\infty, x_0)$.

C. If $f(x)$ has relative minimum point at x_0, then $f(x)$ is increasing on (x_0, ∞).

D. If $a^2 > 3b$, then $f(x)$ has a relative maximum point and a relative minimum point.

E. $f(x)$ may be concave up on $(-\infty, \infty)$ <<<<

42. A hot air balloon rising straight up from a level field is tracked by a range finder 500 feet from the lift off point. At the moment the range finder's angle of elevation is 45°, the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment?

A. 35 ft/min

B. 70 ft/min

C. $70\sqrt{2}$ ft/min

D. 140 ft/min <<<<<

E. 210 ft/min

43. **(Tie Break No 5)** What is the difference between the maximum and minimum values of the function $f(x) = x + 2\sin x$ on the closed interval $[\pi/6, \pi]$?

A. $5\pi/6$

B. $2\pi/3 + \sqrt{3}/2$

C. $\pi/2 + \sqrt{3} - 1$ <<<<<<<<<<<

D. $5\pi/6 - 1$

E. $\pi/3 - \sqrt{3}$

44. Three boxes are each filled with the same number of marbles. Each of 6 empty cans receives one-twelfth of the marbles from each of the three boxes. Each box then contains 12 more marbles than each can. How many marbles were in each box at the beginning?

A. 150

B. 100

C. 48 <<<<<<<<<

D. 36

E. 24

45. A group of applicants for a job consists of 6 men and 5 women. If 3 different applicants are randomly selected from this group, what is the probability of selecting at least one woman applicant?

A. 19/33

B. 29/33 <<<<

C. 4/33

D. 58/165

E. 3/11