1. Let $f(x)=\dfrac{1}{x^2-5}$ and let $g(x)=\sqrt{9-x^2}$. What integers are in the domain of $(f\circ g)(x)$?

Since the domain of g is $[-3,3]$, the only integers to be considered are -3, -2, -1, 0, 1, 2, and 3. $g(-2)=g(2)=\sqrt{5}$, which is not in the domain of f, so -2 and 2 must be excluded. $g(-3)=g(3)=0$, $g(-1)=g(1)=\sqrt{8}$, and $g(0)=3$, all of which are in the domain of f, so the integers of domain of $(f\circ g)(x)$ is $\{-3,-1,0,1,3\}$.

Correct Answer: $-3,-1,0,1,3$

2. Jim completes one-third of a trip at a speed of 20 miles per hour and the remaining two-thirds of the trip at a speed of 60 miles per hour. What was his average speed for the entire trip?

Let d = the total length of the trip. Let t_1 = the time elapsed during the first third of the trip. Then, $t_1=\dfrac{\frac{1}{3}d}{20}=\dfrac{d}{60}$. Let t_2 = the time elapsed during the final two-thirds of the trip. Then, $t_2=\dfrac{\frac{2}{3}d}{60}=\dfrac{d}{90}$. Then, $r=\dfrac{d}{t_1+t_2}=\dfrac{d}{\frac{d}{60}+\frac{d}{90}}=\dfrac{d}{\frac{5d}{180}}=d\left(\dfrac{180}{5d}\right)=36$.

Correct Answer: 36 miles per hour

3. Let p_k denote the k^{th} prime number. So $p_1=2$, $p_2=3$, $p_3=5$, etc. Find $\sum_{k=1}^{7}(i)^{p_k}$.

$$\sum_{k=1}^{7}(i)^{p_k}=i^2+i^3+i^5+i^7+i^{11}+i^{13}+i^{17}=i^2+i^3+i^4i+i^4i^3+\left(i^4\right)^2i^3+\left(i^4\right)^3i+\left(i^4\right)^4i$$
$$=(-1)+(-i)+i+(-i)+(-i)+i+i=-1$$

Correct Answer: -1

4. Find the exact value of $\sin\dfrac{2015\pi}{6}$.

$$\sin\frac{2015\pi}{6}=\sin\left(\frac{11\pi}{6}+\frac{2004\pi}{6}\right)=\sin\left(\frac{11\pi}{6}+334\pi\right)=\sin\left(\frac{11\pi}{6}+167(2\pi)\right)=$$
$$\sin\left(\frac{11\pi}{6}\right)=-\frac{1}{2}$$

Correct Answer: $-\dfrac{1}{2}$

5. If $1+r+r^2+r^3+\ldots=2015$, find the value of r.

So $\frac{1}{1-r}=2015.\ 1=2015-2015r;\ -2015r=-2014;\ r=\frac{2014}{2015}$

Correct Answer: $\frac{2014}{2015}$

6. How many zeros occur at the end of 10! ?

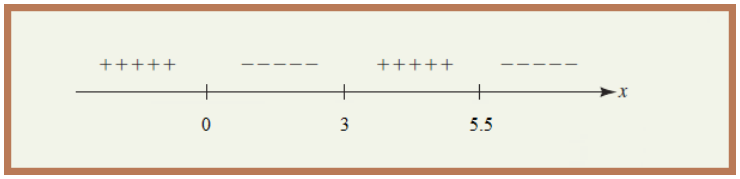
A number is divisible by 10^n but not divisible by 10^{n+1} if and only it is divisible by both 2^n and 5^n but either not divisible by 2^{n+1} or not divisible by 5^{n+1}.
Since $10!=(1)(2)(3)(2^2)(5)(2\cdot3)(7)(2^3)(3^2)(2\cdot5)=2^8 3^4 5^2 7$ is divisible by 2^2 and 5^2 but not divisible by 5^3, it is divisible by 100 but not divisible by 1000.

Therefore there are exactly 2 zeros at the end of 10! = 7,257,600.

Correct Answer: 2

7. What is the greatest integer in the domain of $\log\frac{-2x^2+17x-33}{x}$?

The expression $\frac{-(2x-11)(x-3)}{x}$ may only change sign at $x=0, x=3,$ and $x=5.5.$



The sign line for the inequality is shown to the right. Since $\log\frac{-2x^2+17x-33}{x}$ is defined if and only if $\frac{-2x^2+17x-33}{x}>0,$ its domain is $(-\infty,0)\cup(3,5.5)$. Therefore, the greatest integer in its domain is 5.

Correct Answer: 5

8. How much wood could a wood chuck chuck if a wood chuck could chuck $\log 1000^3 + 6\ln e - \log_\pi \pi^2$ cords of wood?

$= \log\left(10^3\right)^3 + 6(1) - 2 = \log 10^9 + 6 - 2 = 9 + 6 - 2 = 15 - 2 = 13$

Correct Answer: 13

9. Which is the largest of the following $1^{48}, 2^{42}, 3^{36}, 4^{30}, 5^{24}, 6^{18}, 7^{12}, 8^6, 9^0$? (Write the answer in its current exponential form)

$1^{48}, 2^{42}, 3^{36}, 4^{30}, 5^{24}, 6^{18}, 7^{12}, 8^6, 9^0$ is the number with the largest sixth root.

The sixth roots of these numbers are respectively

$1^8 = 1,\ 2^7 = 128,\ 3^6 = 729,\ 4^5 = 1024,\ 5^4 = 625,\ 6^3 = 216,\ 7^2 = 49,\ 8^1 = 8,$ and $9^0 = 1.$

Since 4^{30} has the largest sixth root, it is the largest of these numbers.

Correct Answer: 4^{30}

10. Identify this famous mathematician. c. 287 BC – c. 212 BC

Hint: “Eureka!”

Correct Answer: Archimedes

11. A child has a bag of m&m's with 20 green m&m's, 8 brown m&m's, 12 red m&m's and 10 yellow m&m's. If he randomly selects 3 m&m's and eats them as he pulls them out of the bag, what is the probability that all three m&m's are brown? Write the probability as a fraction.

Let B_1 denote the event "The first m&m is brown."

Let B_2 denote the event "The second m&m is brown."

Let B_3 denote the event "The third m&m is brown."

Then $P(B_1 \cap B_2 \cap B_3) = P(B_1) \cdot P(B_2 \mid B_1) \cdot P(B_3 \mid B_1 \cap B_2)$

$= \frac{8}{50} \cdot \frac{7}{49} \cdot \frac{6}{48} = \frac{4}{25} \cdot \frac{1}{7} \cdot \frac{1}{8} = \frac{1}{350}$

Correct Answer: $\frac{1}{350}$

12. Given that $x = \frac{1}{2}$ radian, what is the value of $\log_5(\tan x) - \log_5(\sin x) + \log_5(\cos x)$?

$\frac{\tan x \cos x}{\sin x} = 1$ so long as $\sin x \neq 0$, which is the case since $x = \frac{1}{2}$.

Correct Answer: 0

13. What is the sum of the digits of $10^{55} - 55$?

$53 \cdot 9 + 4 + 5 = 486$

Correct Answer: 486

14. What is the value of $\sum_{k=1}^{100}(-1)^{k+1}k = 1-2+3-4+5-\ldots+99-100$?

$$= (1-2)+(3-4)+(5-6)+\ldots+(99-100)$$
$$= (-1)+(-1)+\ldots+(-1) \qquad [50\ \textit{terms}]$$
$$= -50$$

Correct Answer: -50

15. Three six-sided dice are rolled. How many outcomes have at least one die showing 5?

There are $6\cdot6\cdot6=216$ total outcomes for the sample space of rolling three six-sided dice. $5\cdot5\cdot5=125$ of these outcomes involve no 5s at all. The difference $216-125=91$ is the number of outcomes that have at least one die showing 5

Correct Answer: 91

16. Find the fourth term in the Binomial Theorem expansion of $(3x-2y)^6$.

The fourth term is $\left({}_6C_3\right)(3x)^3(-2y)^3=\left(\frac{6\cdot5\cdot4}{3\cdot2\cdot1}\right)(27x^3)(-8y^3)=-4320x^3y^3$.

Correct Answer: $-4320x^3y^3$

17. Clean clothes, Inc. (CCI) advertises its product, Clean Wet, (a household cleaning solution), on television each week. After the first ad, CCI sold 100,000 gallons of Clean Wet. Each week thereafter sales dropped by 10 percent. In the long run, if this trend continues indefinitely, what will be the total number of gallons of Clean Wet sold?

$$100000\,[1+0.9+0.9^2+0.9^3+\ldots]=\sum_{n=0}^{\infty}100000\cdot\frac{1}{1-0.9}=1000000$$

Correct Answer: 1,000,000

18. A fair coin is tossed. If a head occurs, 1 fair die is rolled and if a tail occurs then 2 fair dice are rolled. If Y is the total on the die or dice, find P(Y = 6).

Let A be the event that head occurs.

$$P(Y=6)=P(A)P(Y=6\,|\,A)+P(A')P(Y=6\,|\,A')=\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot\frac{5}{36}=\frac{11}{72}$$

Correct Answer: $\frac{11}{72}$

19. Determine the domain of the real-valued function $f(x)=\sqrt{5-\sqrt{x}}$.
Express the domain using interval notation.

$5-\sqrt{x}\geq 0 \Rightarrow 5\geq\sqrt{x} \Rightarrow 0\leq x\leq 25$

Correct Answer: $[0,25]$

20. Identify this famous mathematician. 1912 - 1954

Hint: He cracked the code.

Correct Answer: Alan Turing