Math Bowl 2015 Written Test Solutions

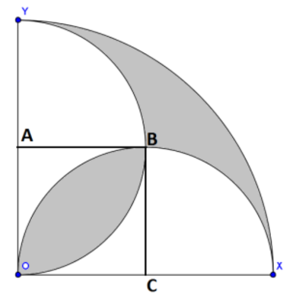
1. Determine "m" so that "$x+3$" is a factor of the polynomial $x^4+3x^3-mx^2-27x+792$.

-3	1	3	-m	-27	792
		-3	0	3m	-9m+81
	1	0	-m	3m-27	-9m+873

$$-9m+873=0, m=\frac{-873}{-9}, m=97$$

Correct Answer: D

2. In the diagram below, XOY is a quarter-circle. Semicircles are drawn with diameters $\overline{OX}$ and $\overline{OY}$ as shown. Find the area of the shaded region given that $XO=6$.



If we consider the expression:
(Area of sector XOY) – (Area of semicircle A) – (Area of semicircle C), this will give us the area of the shaded region bounded by the arc XY minus the shaded region contained in the square OABC. Since the area of the sector XOY is 9π and the area of the semicircles are $\frac{9\pi}{2}$ each.
The expression comes to 0. This implies that the two shaded regions have the same area.
The area of the shaded region contained in the square OABC is
2[(Area of sector OAB) – (Area of triangle OAB)] = $\frac{9}{2}\pi-9$.
The area of the both shaded regions is $2\left(\frac{9}{2}\pi-9\right)=9\pi-18$

Correct Answer: D

3. The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by

$f'(c)=3c^2-4c-1$.

The slope of the line passing through $(0, f(0))$ and $(5, f(5))$ is $\frac{f(5)-f(0)}{5-0}=\frac{71-1}{5}=\frac{70}{5}=14$.

The value of c must then be a solution to $3c^2-4c-1=14$.

$3c^2-4c-15=0$, $(3c+5)(c-3)=0$, $3c+5=0$ or $c-3=0$, $c=-\frac{5}{3}$ or $c=3$

Two points satisfying the given criteria are thus $\left(-\frac{5}{3},-\frac{203}{27}\right)$ and $(3,7)$.

Correct answer: D

4. Determine the sum of the real values of x and y for the following statement to be true.

$$-i(x+3y)+(2x-y+1)=\frac{8}{i}$$

First observe that $\frac{8}{i}=\frac{8i}{i^2}=\frac{8i}{-1}=-8i$.

Set the real parts equal each other and likewise the imaginary parts and solve the resulting system of equations.

$\begin{Bmatrix} 2x-y+1=0 \\ x+3y=8 \end{Bmatrix}$ has solution set $\left\{\left(\frac{5}{7},\frac{17}{7}\right)\right\}$, so $x+y=\frac{5}{7}+\frac{17}{7}=\frac{22}{7}$.

Correct Answer: C

5. Find A such that $0° < A < 90°$ and $\cos 50° - \sin 20° = \sqrt{3}\cos A$

$$\cos 50° - \sin 20° = \sin 40° - \sin 20° = 2\cos 30° \sin 10° = 2\left(\frac{\sqrt{3}}{2}\right)\sin 10° = \sqrt{3}\sin 10°$$

By the Co-function identity $\sin 10° = \cos 80°$. Therefore, $A = 80°$

Correct Answer: E

6. Two equal masses of different types of rocket fuel burn at different, but constant, rates. If the first mass of fuel is consumed in 5 minutes, and the second in 4 minutes, when will the second mass be half the first?

Let m = the original mass of each type of rocket fuel.

Then $m_1=\left(1-\frac{t}{5}\right)m$ and $m_2=\left(1-\frac{t}{4}\right)m$ represent respectively the quantities of each type of fuel that remain after t minutes.

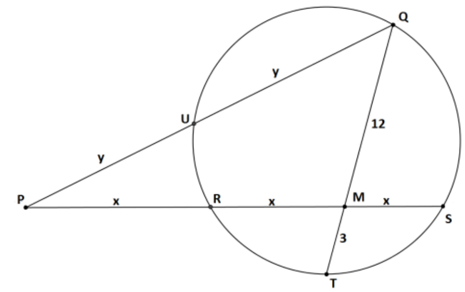
To determine when m_2 is equal to half of m_1, solve the equation $m_2=\frac{1}{2}m_1$.

$\left(1-\frac{t}{4}\right)m=\frac{1}{2}\left(1-\frac{t}{5}\right)m$, $2\left(1-\frac{t}{4}\right)=\left(1-\frac{t}{5}\right)$, $2-\frac{t}{2}=1-\frac{t}{5}$, $1=\frac{3}{10}t$,

$t=\frac{10}{3}$ minutes or 200 seconds

Correct Answer: B

7. In the figure, points R and M trisect $\overline{PS}$. Point U is the midpoint of $\overline{PQ}$, $TM = 2$, and $MQ = 8$. Find PU.



Since $(MT)(MQ)=(RM)(MS)$ we have $(3)(12)=(x)(x)$ or $x=6$.
Since $(PU)(PQ)=(PR)(PS)$ we have

$(y)(2y)=x(3x), 2y^2=3x^2, y=\sqrt{\frac{3}{2}x^2},$

$y=\sqrt{\frac{3}{2}6^2}, y=3\sqrt{6}$

Correct Answer: C

8. A rubber ball dropped 40 feet rebounds on each bounce $\frac{2}{5}$ of the height from which it fell. How far will it travel before coming to rest?

The total distance the ball travels while falling is $40+40\left(\frac{2}{5}\right)+40\left(\frac{2}{5}\right)^2+\ldots$

The total distance the ball travels while rebounding is $40\left(\frac{2}{5}\right)+40\left(\frac{2}{5}\right)^2+40\left(\frac{2}{5}\right)^3+\ldots$

The total distance the ball travels is the sum of these two geometric series, each with ratio $r=\frac{2}{5}$.

The total distance traveled is thus $\frac{40}{1-\frac{2}{5}}+\frac{40\left(\frac{2}{5}\right)}{1-\frac{2}{5}}=40\left(\frac{5}{3}\right)+40\left(\frac{2}{5}\right)\left(\frac{5}{3}\right)=\frac{200}{3}+\frac{80}{3}=\frac{280}{3}$ feet

Correct Answer: A

9. Compute $(\sin 75° - \sin 165°)^4$

$\sin 75° = \sin(30°+45°) = \sin(30°)\cos(45°)+\cos(30°)\sin(45°) = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$

$\sin 165° = \sin(120°+45°) = \sin(120°)\cos(45°)+\cos(120°)\sin(45°) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$

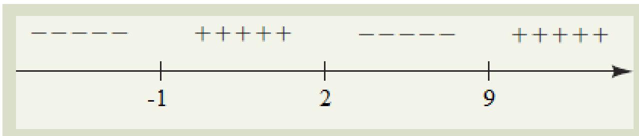
$(\sin 75° - \sin 165°)^4 = \left(\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}-\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right)^4 = \left(\frac{2\sqrt{2}}{4}\right)^4 = \frac{1}{4}$

Correct Answer: B

10. Determine the domain of the function $f(x) = \log_2 \dfrac{x^2 - 8x - 9}{x - 2}$.

The domain of this function is the solution set to the inequality $\dfrac{x^2 - 8x - 9}{x - 2} > 0$.

Factoring the numerator to obtain $\dfrac{(x-9)(x+1)}{x-2}$ and choosing test values, we obtain the following sign graph:



Therefore, the solution set to the inequality and the domain of the function is $(-1, 2) \cup (9, \infty)$.

Correct Answer: B

11. Find the length of the longest chord of the graph of $x^2 + y^2 + 4x + 2y - 20 = 0$ that passes through the point $(-1, 2.5)$.

Rewriting the equation into the standard form of a circle we have:

$$x^2 + 4x + y^2 + 2y = 20$$
$$x^2 + 4x + 4 + y^2 + 2y + 1 = 25$$
$$(x+2)^2 + (y+1)^2 = 25$$

This is a circle with center (-2, -1) and radius 5. The length of the longest cord is the diameter of the circle. Therefore, the longest cord is 10.

Correct Answer: D

12. The temperature on a summer evening can be determined by counting the number of chirps a cricket makes is 15 seconds and adding 38. The speed of a hypothetical ant varies with the temperature in such a way that the number of inches that the ant moves in 12 minutes is 38 less than the temperature. How many times does the cricket chirp while the ant travels 10 inches?

Let T = the temperature; let C = the cricket's chirping frequency (chirps/minute);
Let r = the cricket's rate of speed (inches/minute)

Then $T = \frac{1}{4}C + 38$ and $r = \dfrac{T - 38}{12}$. Substituting $\frac{1}{4}C + 38$ for T in the latter equation, we have $r = \dfrac{\frac{1}{4}C + 38 - 38}{12} = \dfrac{1}{48}C$. So, $C = 48r$. So, the number of chirps in any span of time is 48 times the number of inches traveled in any span of time. Therefore, the number of times the cricket chirps in when the ant travels 10 inches is $48(10) = 480$.

Correct Answer: A

13. If the roots of the equation $x^2 + bx + c = 0$ are r and s, the value of $\frac{(r+s)^2}{rs}$ in terms of b And c is

Applying the quadratic formula, the roots of the given equation are

$$r = \frac{-b+\sqrt{b^2-4c}}{2} \; and \; s = \frac{-b-\sqrt{b^2-4c}}{2}.$$

$$\frac{(r+s)^2}{rs} = \frac{\left(\frac{-b+\sqrt{b^2-4c}}{2} + \frac{-b-\sqrt{b^2-4c}}{2}\right)^2}{\left(\frac{-b+\sqrt{b^2-4c}}{2}\right)\left(\frac{-b-\sqrt{b^2-4c}}{2}\right)} = \frac{\left(\frac{-b}{2}+\frac{-b}{2}\right)^2}{\left(\frac{(-b)^2-\left(b^2-4c\right)}{4}\right)} = \frac{b^2}{\frac{4c}{4}} = \frac{b^2}{c}$$

Correct Answer: C

14. If $\sin 2x = \frac{12}{13}$, find the value of $\sin^4 x + \cos^4 x$

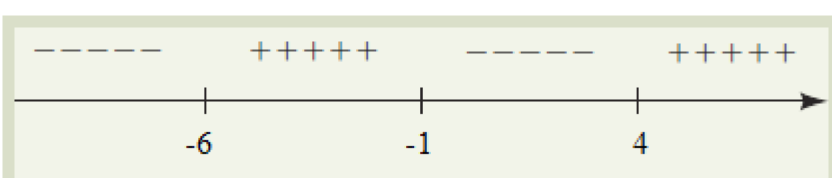
$\sin 2x = \frac{12}{13}$ implies $2\sin x \cos x = \frac{12}{13}$ or $\sin x \cos x = \frac{6}{13}$. $\sin^4 x + \cos^4 x$ can be obtained by squaring $\sin^2 x + \cos^2 x = 1$. We have $\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x = 1$ or $\sin^4 x + \cos^4 x = 1 - 2\sin^2 x\cos^2 x$. Substituting our given information we have

$$\sin^4 x + \cos^4 x = 1 - 2\sin^2 x\cos^2 x = 1 - 2\left(\frac{6}{13}\right)^2 = 1 - 2\left(\frac{36}{169}\right) = 1 - \frac{72}{169} = \frac{97}{169}$$

Correct Answer: D

15. Solve the rational inequality $\frac{2}{x-4} \geq \frac{1}{x+1}$.

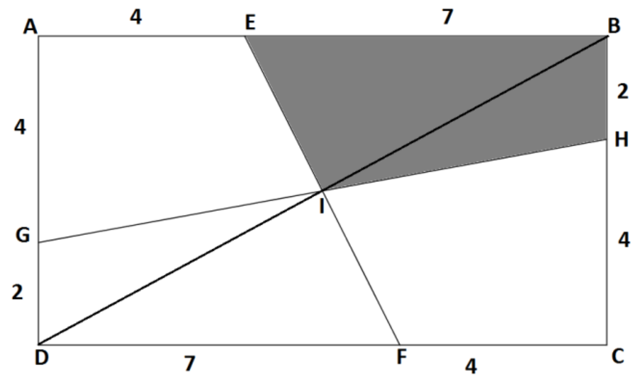
$$\frac{2}{x-4} - \frac{1}{x+1} \geq 0, \quad \frac{x+6}{(x-4)(x+1)} \geq 0$$



The solution set is $[-6,-1) \cup (4,\infty)$

Correct Answer: A

16. The lengths indicated on the rectangle shown are in inches. What is the number of square inches in the area of the shaded region?



Drawing a diagonal from D to B splits the shaded region into two triangles.

Area of Triangle EIB =
$\frac{1}{2}(EB)\left(\frac{DG+GA}{2}\right)=\frac{1}{2}(7)\left(\frac{2+4}{2}\right)=\frac{21}{2}$

Area of Triangle BIH =
$\frac{1}{2}(BH)\left(\frac{DF+FC}{2}\right)=\frac{1}{2}(2)\left(\frac{7+4}{2}\right)=\frac{11}{2}$

The area of the shaded region is
$\frac{21}{2}+\frac{11}{2}=16$

Correct Answer: C

17. Find the sum of the first ten terms in the arithmetic sequence $3x+1,\ 2x+4,\ -2x-2,\ldots$

$S_n=\frac{n}{2}(a_1+a_n)$. $d_1=(2x+4)-(3x+1)=-x+3$. $d_2=(-2x-2)-(2x+4)=-4x-6$.
Since $d_1=d_2$, $-x+3=-4x-6$, $3x=-9$, $x=-3$. $d=-(-3)+3=6$ *or* $d=-4(-3)-6=6$.
$a_n=a_1+(n-1)d$, $a_n=-8+(n-1)(6)$, $a_n=6n-14$, $a_{10}=6(10)-14=46$,
$S_n=\frac{n}{2}(a_1+a_n)$, $S_n=\frac{10}{2}(-8+46)=190$

Correct Answer: D

18. Find the value of $\tan x$ if $\sin x+\cos x=\frac{7}{13}$ and $\pi/2<x<\pi$

Squaring the equation gives us
$\sin^2 x+2\sin x\cos x+\cos^2 x=\frac{49}{169}$ or $2\sin x\cos x=-\frac{120}{169}$ or $\sin x\cos x=-\frac{60}{169}$.

By making a substitution for the given equation for cos *x*, we have
$\sin x\left(\frac{7}{13}-\sin x\right)=-\frac{60}{169}$ or as a quadratic equation $\sin^2 x-\frac{7}{13}\sin x-\frac{60}{169}=0$.

Solving the equation we have $(13\sin x+5)(13\sin x-12)=0$ or $\sin x=-\frac{5}{13},\frac{12}{13}$.

Since $\pi/2<x<\pi$, we choose $\sin x=\frac{12}{13}$. By drawing the triangle, $\tan x=-\frac{12}{5}$.

Correct Answer: C

19. Find the constant term of a polynomial of degree 4 with rational coefficients that has a leading coefficient of 1 and two roots equal to $2-i$ and $2+\sqrt{3}$.

Since all coefficients are real and $2-i$ is a root, $2+i$ is also a root.
Since all coefficients are rational and $2+\sqrt{3}$ is a root, $2-\sqrt{3}$ is also a root.
Since the leading coefficient is 1, the constant term is the product of the roots.
$(2-i)(2+i)(2+\sqrt{3})(2-\sqrt{3}) = (4+1)(4-3) = 5(1) = 5$

Correct Answer: B

20. A given polygon has 44 diagonals. How many sides does the polygon have?

A polygon with *n* sides has $\frac{n(n-3)}{2}$ diagonals. Using this formula we have
$\frac{n(n-3)}{2} = 44, n(n-3) = 88, n^2 - 3n = 88, n^2 - 3n - 88 = 0, (n-11)(n+8) = 0, n = -8, 11$
Since the number of sides has to be positive, the polygon has 11 sides.

Correct Answer: C

21. Four fair coins are flipped. Your friend looks at them and observes that **at least one** of them have turned up heads. With this given information, find the probability that **exactly three** of the coins have turned up heads.

$P(\text{exactly 3H}) = \frac{C(4,3)}{2^4} = \frac{4}{16} = \frac{1}{4}$. $\quad P(\text{at least 1H}) = 1 - P(\text{0H}) = 1 - \frac{1}{2^4} = \frac{15}{16}$.

$$P(\text{exactly 3H}|\text{at least 1H}) = \frac{P(\text{exactly 3H and at least 1H})}{P(\text{at least 1H})} = \frac{P(\text{exactly 3H})}{P(\text{at least 1H})} = \frac{\frac{4}{16}}{\frac{15}{16}} = \frac{4}{15}$$

Correct answer: D

22. Find x^2 if $\sqrt[3]{x+9}-\sqrt[3]{x-9}=3.$

$\sqrt[3]{x+9}-\sqrt[3]{x-9}=3$, $\left(\sqrt[3]{x+9}-\sqrt[3]{x-9}\right)^3=(3)^3$

$x+9-3\left(\sqrt[3]{x+9}\right)^2\left(\sqrt[3]{x-9}\right)+3\left(\sqrt[3]{x+9}\right)\left(\sqrt[3]{x-9}\right)^2-(x-9)=27$,

$18-3\sqrt[3]{x+9}\sqrt[3]{x-9}\left(\sqrt[3]{x+9}-\sqrt[3]{x-9}\right)=27$

$-3\sqrt[3]{x+9}\sqrt[3]{x-9}\left(\sqrt[3]{x+9}-\sqrt[3]{x-9}\right)=9$, $-3\sqrt[3]{x^2-81}(3)=9$, $\sqrt[3]{x^2-81}=-1$, $x^2-81=-1$

$x^2=80$

Correct Answer: D

23. Compute $\sin\left(2\cos^{-1}\left(\frac{\sqrt{5}}{5}\right)\right)$

$$\sin\left(2\cos^{-1}\left(\frac{\sqrt{5}}{5}\right)\right)=2\sin\left(\cos^{-1}\frac{\sqrt{5}}{5}\right)\cos\left(\cos^{-1}\frac{\sqrt{5}}{5}\right)=2\left(\frac{\sqrt{5}}{5}\right)\left(\frac{2\sqrt{5}}{5}\right)=\frac{4}{5}$$

Correct Answer: D

24. A can of oil is $\frac{4}{5}$ full. If 6 bottles of oil are poured out of it, it will be $\frac{3}{4}$ full.

How many bottles of oil can the can hold?

Let x = the number of bottles a full can can hold.

Then $\frac{4}{5}x-6=\frac{3}{4}x$.

$16x-120=15x$

$x=120$ bottles of oil

Correct Answer: A

25. The minute hand on a clock sweeps out 20 minutes. If the hand is 4cm long creating the central angle $\angle AOB$, what is the area of the region swept out by the hand excluding the area of the isosceles triangle, ΔAOB?

The area of the sector made by the minute hand is $A=\frac{1}{2}(4)^2\pi\left(\frac{120°}{360°}\right)=\frac{16\pi}{3}$.

The area of the isosceles triangle is $A=\frac{1}{2}\left(4\sqrt{3}\right)(2)=4\sqrt{3}$.

The area made by the sector excluding the isosceles triangle is $\frac{16}{3}\pi-4\sqrt{3}$.

Correct Answer: E

26. Given $f(x)$ such that $f(1-x)+(1-x)f(x)=5,$ find $f(5).$

Let $t=1-x$.
Then $f(t)=tf(1-t)=5$, $f(t)+t[5-(1-t)f(t)]=5$, $f(t)=5t-t(1-t)f(t)=5$,

$f(t)[1-t(1-t)]=5-5t$, $f(t)=\frac{5(1-t)}{1-t(1-t)}$, $f(5)=\frac{5(-4)}{1-5(-4)}=-\frac{20}{21}$

Correct Answer: E

27. Evaluate $\cos\left(\arcsin\frac{4}{5}-\arctan 2\right)$.

Let $A=\arcsin\frac{4}{5}$ and $B=\arctan 2$. Then $\sin A=\frac{4}{5}, \cos A=\frac{3}{5}, \sin B=\frac{2\sqrt{5}}{5}$, and $\cos B=\frac{\sqrt{5}}{5}$

$\cos(A-B)=\cos A\cos B+\sin A\sin B=\left(\frac{3}{5}\right)\left(\frac{\sqrt{5}}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{2\sqrt{5}}{5}\right)=\frac{11\sqrt{5}}{25}$

Correct Answer: D

28. Each valve A, B, and C, when open, releases water into a tank at its own constant rate. With all three valves open, the tank fills in 1 hour, with only valves A and C open, it takes 1.5 hours, and with only valves B and C open, it takes 2 hours. How long will it take to fill the tank with only valves A and B open?

Let a = time for A to fill. Let b = time for B to fill. Let c = time for C to fill

$$\begin{Bmatrix} \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1 \\ \frac{1}{a}+\frac{1}{c}=\frac{2}{3} \\ \frac{1}{b}+\frac{1}{c}=\frac{1}{2} \end{Bmatrix}, \ \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{c}=\frac{2}{3}+\frac{1}{2}, 1+\frac{1}{c}=\frac{7}{6}, \ \frac{1}{c}=\frac{1}{6}, c=6, \frac{1}{a}=\frac{2}{3}-\frac{1}{6}=\frac{1}{2}, a=2$$

$$\frac{1}{b}=\frac{1}{2}-\frac{1}{6}=\frac{1}{3}, b=3$$

With only valves A and B open, the tank fills in $\frac{1}{\frac{1}{a}+\frac{1}{b}}=\frac{1}{\frac{1}{2}+\frac{1}{3}}=\frac{1}{\frac{5}{6}}=\frac{6}{5}$ hours.

Correct Answer: B

29. Two chords of a circle, $\overline{UP}$ and $\overline{TL}$, intersect at A. If $LA=5cm$, $LT=13cm$, $UP=14cm$ and $AU \le UP$, find AU.

From the information $(AU)(AP)=(AT)(AL)$.Since $(AT)(AL)=LT$, $AT=8$. Let $x=AU$, then $AP=14-x$. We now have: $(x)(14-x)=(8)(5)$.

Solving this equation gives $x=4,10$. Since $AU \le UP$, x = 4

Correct Answer: C

30. Let C_m, C_f, P_m, and P_f denote respectively the numbers of male chemistry majors, female chemistry majors, male physics majors, and female physics majors.

$C_m = 2P_m$ since there are twice as many male chemistry majors as male physics majors. $P_f = 3P_m$ since there are three times as many female physics majors as male physics majors. $P_f = C_f$ since half of the females in the class are chemistry majors and the other half physics majors. $P_m + P_f + C_m + C_f = 72$ since there are 72 people in the class with no one majoring in both physics and chemistry.

$P_m + 3P_m + 2P_m + 3P_m = 72, 9P_m = 72, P_m = 8$

Correct Answer: C

31. Let $x=\sqrt{5-12i}$. Evaluate $2x+3i$.

$x=\sqrt{5-12i}=a+bi$, $5-12i=\left(a+bi\right)^2$, $5-12i=a^2+2abi-b^2$, $a^2-b^2=5$ *and* $2ab=-12$,

$b=-\dfrac{6}{a}$, $a^2-\left(-\dfrac{6}{a}\right)^2=5$, $a^2-\dfrac{36}{a^2}=5$, $a^4-5a^2-36=0$, $(a^2-9)(a^2+4)=0$, $a=\pm 3$.

If $a=3$ then $b=-2$. If $a=-3$ then $b=2$. So $x=3-2i$ or $x=-3+2i$.

If $x=3-2i$, then $2(3-2i)+3i=6-4i+3i=6-i$. If $x=-3+2i$ then $2(-3+2i)+3i=-6+4i+3i=-6+7i$.

Correct Answer: D

32. Find c^2 for triangle *ABC* if *a* = 25 in., *b* = 32 in., and *C* = 60°.

Applying the Law of Cosines we have

$c^2=a^2+b^2-2ab\cos C$

$=\left(25\right)^2+\left(32\right)^2-2\left(25\right)\left(32\right)\cos\left(60^\circ\right)$

$=849$

Correct Answer: C

33. The sum of the solutions of $\left(\log_4 x\right)^2=\log_4(x^2)$ is equal to …

$y=\log_4 x$

$y^2=2y$

$y^2-2y=0$

$y(y-2)=0$

$y=0$ *or* $y=2$

$\log_4 x=0$ *or* $\log_4 x=2$

$4^0=x$ *or* $4^2=x$

$x=1$ *or* $x=16$ The sum of the solutions is 17. Correct Answer E

34. For cyclic rectangle $ABCD$, $AB=8$. Diagonal $\overline{DB}$ of the rectangle contains the center of the circle and $DB=10$. Find the perimeter of $ABCD$.

With diagonal place in the rectangle, a right triangle, ABD is formed. By the Pythagorean Theorem, AD = 6. The perimeter of the rectangle is $8+6+8+6=28$.

Correct Answer: A

35. Find an equation of the ellipse with center (1, 2), vertex (1, 4) and containing the point $\left(\frac{1}{2}, 2+\sqrt{3}\right)$.

Since the *x*-coordinates of the center and vertices are the same, the major axis of the ellipse is vertical.

So, the equation of the ellipse can be written in the form $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

Since the center is (1,2), the equation is $\frac{(x-1)^2}{b^2}+\frac{(y-2)^2}{a^2}=1$

Since the distance between the center and a vertex is 2, we have $a=2$.

So, the equation is $\frac{(x-1)^2}{b^2}+\frac{(y-2)^2}{2^2}=1$

Finally, since the ellipse passes through the point $\left(\frac{1}{2}, 2+\sqrt{3}\right)$, we have

$$\frac{\left(\frac{1}{2}-1\right)^2}{b^2}+\frac{(2+\sqrt{3}-2)^2}{2^2}=1,\ \frac{1}{4b^2}+\frac{3}{4}=1, 1+3b^2=4b^2, b^2=1$$

So, the equation is $\frac{(x-1)^2}{1}+\frac{(y-2)^2}{4}=1$

Correct Answer: C

36. Which of the following is an identity?

$$\frac{2\tan x}{1+\tan^2 x}=\frac{2\tan x}{\sec^2 x}=2\tan x\cos^2 x=2\frac{\sin x}{\cos x}\cos^2 x=2\sin x\cos x=\sin 2x$$

Correct Answer: E

37. Solve the system $\begin{cases} 2y^2 - 3xy + 6y + 2x + 4 = 0 \\ 2x - 3y + 4 = 0 \end{cases}$.

$x = \frac{3}{2}y - 2$, $2y^2 - 3\left(\frac{3}{2}y - 2\right)y + 6y + 2\left(\frac{3}{2}y - 2\right) + 4 = 0$,

$2y^2 - \frac{9}{2}y^2 + 6y + 6y + 3y - 4 + 4 = 0$, $-\frac{5}{2}y^2 + 15y = 0$, $-\frac{5}{2}y(y-6) = 0$

$y = 0$ *or* $y = 6$

If $y = 0$ *then* $x = \frac{3}{2}(0) - 2 = -2$. If $y = 6$ *then* $x = \frac{3}{2}(6) - 2 = 7$.

So, the solution set is $\{(-2,0),(7,6)\}$

Correct Answer: A

38. In trapezoid $ABCD$, $\overline{BC} \parallel \overline{AD}$ and M and N are the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively. Find MN if $BC = 2x$, $MN = 3x - 5$, and $AD = 2x + 10$.

Since MN is made from the midpoints of AB and CD, we have $MN = \frac{BC + AD}{2}$.

Thus, $3x - 5 = \frac{(2x) + (2x + 10)}{2}$. Solving the equation gives us $x = 10$. Therefore, $MN = 3(10) - 5 = 25$

Correct Answer: D

39. Let B_k, k = 1, 2, 3 be the event that the box *k* is selected. Since P(B_k) = ck, where c is a constant such that c(1) + c(2) + c(3) =1 implies c = 1/6

E be the event that two marbles drawn are of different colors then P (E / B_1) = $\frac{{}_1C_1 \cdot {}_4C_1}{{}_5C_2} = \frac{4}{10}$

P (E / B_2) = $\frac{{}_2C_1 \cdot {}_3C_1}{{}_5C_2} = \frac{6}{10}$ and P (E / B_3) = $\frac{{}_3C_1 \cdot {}_2C_1}{{}_5C_2} = \frac{6}{10}$

Using the total probability rule, P(E) = $\sum_{k=1}^{3} P(B_k) \cdot P(E/B_k) = \frac{1}{6} \cdot \frac{4}{10} + \frac{2}{6} \cdot \frac{6}{10} + \frac{3}{6} \cdot \frac{6}{10} = \frac{17}{30}$

Correct Answer: A

40. Initially two trains are 340 miles apart. They start moving toward each other on parallel tracks. The speed of one train is 5 mph faster than the speed of the other train. At the end of two hours, they are 85 miles apart after passing each other. At what time did they pass each other?

Let x = the speed of the slower train. Then x + 5 = the speed of the faster train.
Let t = the time when they pass each other.
Then $xt+(x+5)t=$ the sum of the distances traveled by both trains = 340 miles.

$xt+(x+5)t=340$, $t(2x+5)=340$
At the end of two hours,
$x(2-t)+(2-t)(x+5)=85$, $(2-t)(2x+5)=85$, $(2-t)\left(\frac{340}{t}\right)=85$, $680-340t=85t$,

$680=425t$, $t=\frac{680}{425}=1.6$

Correct Answer: B

41. The height of a rider on a Ferris wheel can be modeled by the function $h(t)=k-A\cos(xt)$ where t is measured in minutes and h in feet. If the height of the rider alternates between 10 ft and 80 ft and the rider completes one revolution every 4 minutes, find a formula for h.

$k=\frac{80+10}{2}=45$, $A=\frac{80-10}{2}=35$ $4=\frac{2\pi}{x}\rightarrow x=\frac{\pi}{2}$. Therefore the formula is
$h(t)=45-35\cos\left(\frac{\pi}{2}t\right)$

Correct Answer: D

42. Find f^{-1} for the function $f(x)=5\cdot 2^{x-3}+4$.

$y=5\cdot 2^{x-3}+4$, $x=5\cdot 2^{y-3}+4$, $x-4=5\cdot 2^{y-3}$, $\frac{x-4}{5}=2^{y-3}$, $\log_2\left(\frac{x-4}{5}\right)=y-3$,
$y=\log_2\left(\frac{x-4}{5}\right)+3$, $f^{-1}(x)=\log_2\left(\frac{x-4}{5}\right)+3$

Correct Answer: B

43. Find the number of sides in a polygon whose interior angles sum to 2160°.

The sum of measures of the interior angles of a polygon with n sides is $(n-2)180°$.
Using this formula we have, $(n-2)180° = 2160°$. Solving this equation we have $n = 14$.

Correct Answer: B

44. Solve the determinant equation $\begin{vmatrix} x & 1 & 2 \\ 1 & x & 3 \\ 0 & 1 & 2 \end{vmatrix} = -4x$

Expand by cofactors about the first column.

$$x(2x-3)-1(2-2)+0(3-2x)=-4x$$
$$2x^2-3x=-4x$$
$$2x^2+x=0$$
$$x(2x+1)=0$$
$$x=0 \ \ or \ \ x=-\frac{1}{2}$$

Correct Answer: D

45. For $\sin\theta = 2\cot\theta$, find the value(s) of $\cos\theta$.

$$\sin\theta = 2\cot\theta, \sin\theta = 2\frac{\cos\theta}{\sin\theta}, \sin^2\theta = 2\cos\theta, 1-\cos^2\theta = 2\cos\theta, 0 = \cos^2\theta + 2\cos\theta - 1$$

Solving the quadratic equation we have $\cos\theta = \dfrac{-2 \pm \sqrt{4-4(1)(-1)}}{2} = \dfrac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

Correct Answer: B