Math Bowl Written Test 2015

1. Determine "m" so that "$x+3$" is a factor of the polynomial $x^4+3x^3-mx^2-27x+792$.

   A. 113

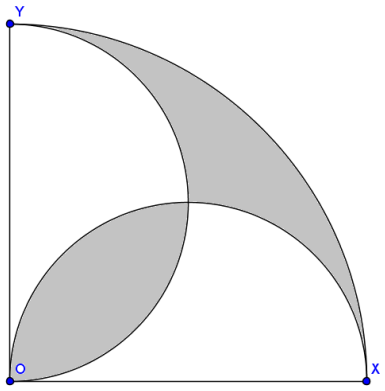
   B. 77

   C. 61

   D. 97

   E. None of the above

2. In the diagram below, XOY is a quarter-circle. Semicircles are drawn with diameters $\overline{OX}$ and $\overline{OY}$ as shown. Find the area of the shaded region given that $XO=6$.



   A. $\frac{9}{4}\pi-\frac{9}{2}$

   B. $9\pi-\frac{9}{2}$

   C. $\frac{9}{2}\pi-9$

   D. $9\pi-18$

   E. $18\pi-36$

3. Let $f(x)=x^3-2x^2-x+1$. Find a point on the graph of f where the tangent line is parallel to the line passing through the points $(0, f(0))$ and $(5, f(5))$.

   A. (0, 1)

   B. (2, -1)

   C. $\left(2.5, \frac{13}{8}\right)$

   D. (3, 7)

   E. (4, 29)

4. Determine the sum of the real values of x and y for the following statement to be true:

$$-i(x+3y)+(2x-y+1)=\frac{8}{i}$$

A. $\frac{3}{4}$

B. $-\frac{11}{3}$

C. $\frac{22}{7}$

D. $\frac{12}{5}$

E. None of the above.

5. Find A such that $0° < A < 90°$ and $\cos 50° - \sin 20° = \sqrt{3}\cos A$

A. 10°

B. 20°

C. 40°

D. 70°

E. 80°

6. Two equal masses of different types of rocket fuel burn at different, but constant, rates. If the first mass of fuel is consumed in 5 minutes, and the second in 4 minutes, when will the second mass be half the first?

A. 180 seconds

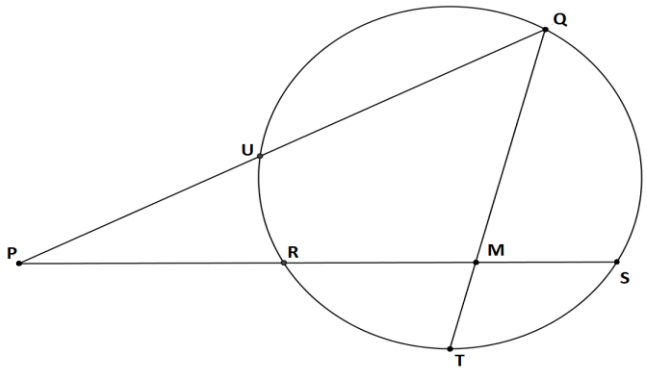
B. 200 seconds

C. 220 seconds

D. 160 seconds

E. 140 seconds

7. In the figure, points R and M trisect $\overline{PS}$. Point U is the midpoint of $\overline{PQ}$, $TM = 3$, and $MQ = 12$. Find PU.



A. 6

B. $9\sqrt{6}$

C. $3\sqrt{6}$

D. 4

E. $5\sqrt{6}$

8. A rubber ball dropped from a height of40 feet rebounds on each bounce $\frac{2}{5}$ of the height from which it fell. How far will it travel before coming to rest?

A. $\frac{280}{3}$ feet

B. $\frac{200}{3}$ feet

C. 80 feet

D. 110 feet

E. 90 feet

9. Compute $\left(\sin 75° - \sin 165°\right)^4$.

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{\sqrt{6}}{2}$

D. $\frac{3}{2}$

E. $\frac{9}{4}$

10. Determine the domain of the function $f(x) = \log_2 \dfrac{x^2 - 8x - 9}{x - 2}$.

A. $(-1, 2) \cup (9, \infty)$

B. $(-\infty, -1) \cup (2, 9)$

C. $(-\infty, -1) \cup (9, \infty)$

D. $(-1, 2) \cup (2, 9)$

E. None of the above

11. Find the length of the longest chord of the graph of $x^2 + y^2 + 4x + 2y - 20 = 0$ containing the point $(-1, 2.5)$.

A. $\dfrac{\sqrt{13}}{2}$

B. $\sqrt{13}$

C. 5

D. 10

E. 13

12. The temperature on a summer evening can be determined as 38 more than the number of chirps made by a cricket in 15 seconds. The speed of a hypothetical ant varies with the temperature in such a way that the number of inches that the ant moves in 12 minutes is 38 less than the temperature. How many times does the cricket chirp while the ant travels 10 inches?

A. 480

B. 380

C. 100

D. 240

E. 250

13. If the roots of the equation $x^2 + bx + c = 0$ are r and s, find the value of $\frac{(r+s)^2}{rs}$ in terms of b and c.

A. $\frac{b^2}{c^2}$

B. $\frac{c^2}{b}$

C. $\frac{b^2}{c}$

D. $-\frac{b}{c}$

E. None of the above

14. If $\sin 2x = \frac{12}{13}$, find the value of $\sin^4 x + \cos^4 x$.

A. $\frac{7}{13}$

B. $\frac{25}{169}$

C. $\frac{36}{169}$

D. $\frac{97}{169}$

E. $\frac{120}{169}$

15. Solve the rational inequality $\frac{2}{x-4} \geq \frac{1}{x+1}$.

A. $[-6,-1) \cup (4,\infty)$

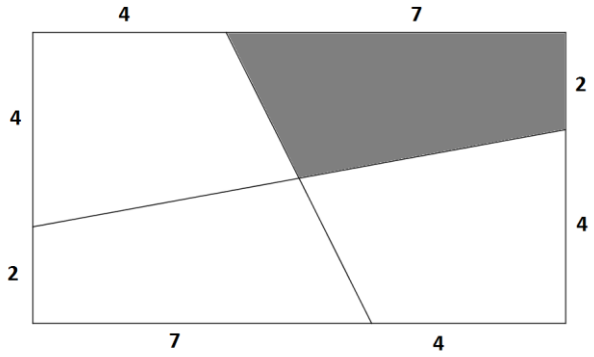
B. $(-\infty,-6] \cup (-1,4)$

C. $(-\infty,-1) \cup (4,\infty)$

D. $[-6,4)$

E. None of the above.

16. **(Tie Break No 1)** The lengths indicated on the rectangle shown are in inches. What is the number of square inches in the area of the shaded region?



A. 14

B. 15

C. 16

D. 16.5

E. 17

17. Find the sum of the first ten terms in the arithmetic sequence $3x+1,\ 2x+4,\ -2x-2,\ldots$.

A. 210

B. -175

C. -235

D. 190

E. -115

18. Find the value of $\tan x$ if $\sin x+\cos x=\frac{7}{13}$ and $\frac{\pi}{2}<x<\pi$.

A. $\frac{12}{13}$

B. $-\frac{5}{13}$

C. $-\frac{12}{5}$

D. $-\frac{5}{12}$

E. $-\frac{13}{5}$

19. Find the constant term of a polynomial of degree 4 with rational coefficients that has a leading coefficient of 1 and two roots equal to $2-i$ and $2+\sqrt{3}$.

A. 4

B. 5

C. 1

D. -1

E. -5

20. A given polygon has 44 diagonals. How many sides does the polygon have?

A. 9

B. 10

C. 11

D. 12

E. 13

21. Four fair coins are flipped. Your friend looks at them and observes that at least one of them have turned up heads. With this given information, find the probability that exactly three of the coins have turned up heads.

A. $\frac{1}{5}$

B. $\frac{1}{4}$

C. $\frac{3}{16}$

D. $\frac{4}{15}$

E. $\frac{5}{16}$

22. Find x^2 if $\sqrt[3]{x+9}-\sqrt[3]{x-9}=3$.

A. 0

B. 82

C. 56

D. 80

E. Not enough information

23. Compute $\sin\left(2\cos^{-1}\left(\frac{\sqrt{5}}{5}\right)\right)$.

A. $\frac{2\sqrt{5}}{5}$

B. $\frac{4\sqrt{5}}{5}$

C. $\frac{2}{5}$

D. $\frac{4}{5}$

E. 2

24. A can of oil is $\frac{4}{5}$ full. If 6 bottles of oil are poured out of it, it will be $\frac{3}{4}$ full. How many bottles of oil can the can hold?

A. 120

B. 100

C. 110

D. 130

E. 90

25. The minute hand on a clock sweeps out 20 minutes. If the hand is 4cm long creating the central angle $\angle AOB$, what is the area of the region swept out by the hand excluding the area of the isosceles triangle, ΔAOB?

A. $\left(\frac{8\pi - 2\sqrt{3}}{3}\right) cm^2$

B. $\left(\frac{8\pi}{3} - 2\sqrt{3}\right) cm^2$

C. $\left(\frac{8\pi}{3} - 4\sqrt{3}\right) cm^2$

D. $\left(\frac{16\pi}{3} - 2\sqrt{3}\right) cm^2$

E. $\left(\frac{16\pi}{3} - 4\sqrt{3}\right) cm^2$

26. **(Tie Break No 2)** Given $f(x)$ such that $f(1-x)+(1-x)f(x)=5,$ find $f(5).$

A. $-\frac{20}{19}$

B. $\frac{20}{21}$

C. 0

D. 1

E. $-\frac{20}{21}$

27. Evaluate $\cos\left(\arcsin\frac{4}{5} - \arctan 2\right).$

A. $\frac{2\sqrt{5}}{25}$

B. $\frac{-\sqrt{5}}{5}$

C. $\frac{2\sqrt{5}}{5}$

D. $\frac{11\sqrt{5}}{25}$

E. $\frac{11\sqrt{5}}{5}$

28. Each valve A, B, and C, when open, releases water into a tank at its own constant rate. With all three valves open, the tank fills in 1 hour, with only valves A and C open, it takes 1.5 hours, and with only valves B and C open, it takes 2 hours. How long will it take to fill the tank with only valves A and B open?

A. $\frac{2}{3}$ hr

B. $\frac{6}{5}$ hr

C. $\frac{3}{2}$ hr

D. $\frac{3}{4}$ hr

E. $\frac{5}{6}$ hr

29. Two chords of a circle, $\overline{UP}$ and $\overline{TL}$, intersect at A. If $LA = 5cm$, $LT = 13cm$, $UP = 14cm$ and $AU \leq UP$, find AU.

A. 10 cm

B. 7 cm

C. 4 cm

D. 5 cm

E. 9 cm

30. A certain college class is taken only by physics and chemistry majors. There are twice as many male chemistry majors as there are male physics majors. Of the physics majors in the class, there are three times as many females as there are males. Half of the females in the class are chemistry majors. If there are 72 people in the class, how many are male physics majors?

A. 6

B. 9

C. 8

D. 16

E. 36

31. **(Tie Break No 3)** Let $x = \sqrt{5-12i}$. Evaluate $2x+3i$.

A. 13

B. $3+2i$

C. $-3-2i$

D. $6-i$

E. Not defined

32. Find c^2 for triangle ABC if a = 25 in., b = 32 in., and C = 60°.

A. 29 in

B. 49 in

C. 849 in

D. 900 in

E. 1249 in

33. Find the sum of the solutions of $(\log_4 x)^2 = \log_4(x^2)$.

A. 2

B. 3

C. 12

D. 16

E. 17

34. For cyclic rectangle $ABCD$, $AB=8$. Diagonal $\overline{DB}$ of the rectangle contains the center of the circle and $DB=10$. Find the perimeter of $ABCD$.

A. 28

B. 48

C. 36

D. 80

E. 60

35. Find an equation of the ellipse with center (1, 2), vertex (1, 4) and containing the point $\left(\frac{1}{2}, 2+\sqrt{3}\right)$.

A. $\frac{(x+1)^2}{1}+\frac{(y+2)^2}{4}=1$

B. $\frac{(x-1)^2}{4}+\frac{(y-2)^2}{1}=1$

C. $\frac{(x-1)^2}{1}+\frac{(y-2)^2}{4}=1$

D. $\frac{(x-1)^2}{4}+\frac{(y-2)^2}{1}=1$

E. There is no such ellipse.

36. Which of the following is an identity?

A. $\frac{2\tan x}{1+\tan^2 x}=\tan 2x$

B. $\frac{2\tan x}{1+\tan^2 x}=\cos 2x$

C. $\frac{2\tan x}{1+\tan^2 x}=\csc 2x$

D. $\frac{2\tan x}{1+\tan^2 x}=\sec 2x$

E. $\frac{2\tan x}{1+\tan^2 x}=\sin 2x$

37. **(Tie Break No 4)** Find the sum of the *x*-coordinates of all the solutions to the system. $\begin{cases} 2y^2 - 3xy + 6y + 2x + 4 = 0 \\ 2x - 3y + 4 = 0 \end{cases}$.

A. 5

B. 2

C. 6

D. 0

E. 4

38. In trapezoid $ABCD$, $\overline{BC} \| \overline{AD}$ and M and N are the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively. Find MN if $BC = 2x$, $MN = 3x - 5$, and $AD = 2x + 10$.

A. 10

B. 15

C. 20

D. 25

E. 30

39. Three boxes are numbered 1, 2, and 3. For k = 1, 2, 3, box k contains k blue marbles and 5 – k red marbles. In a two-step experiment, a box is selected and 2 marbles are drawn from it without replacement. If the probability of selecting box k is equal to ck for some constant c, what is the probability that the 2 marbles drawn have different colors?

A. $\frac{17}{30}$

B. $\frac{34}{75}$

C. $\frac{8}{15}$

D. $\frac{4}{15}$

E. $\frac{1}{2}$

40. **(Tie Break No 5)** Initially two trains are 340 miles apart. They start moving toward each other on parallel tracks. The speed of one train is 5 mph faster than the speed of the other train. At the end of two hours, they are 85 miles apart after passing each other. At what time did they pass each other?

A. 1.4 hours

B. 1.6 hours

C. 1.5 hours

D. 1.2 hours

E. Not enough information

41. The height of a rider on a Ferris wheel can be modeled by the function $h(t) = k - A\cos(xt)$ where *t* is measured in minutes and *h* in feet. If the height of the rider alternates between 10 ft and 80 ft and the rider completes one revolution every 4 minutes, find a formula for *h.*

A. $h(t) = 45 - 40\cos(4t)$

B. $h(t) = 45 - 40\cos\left(\frac{\pi}{2}t\right)$

C. $h(t) = 45 - 35\cos(4t)$

D. $h(t) = 45 - 35\cos\left(\frac{\pi}{2}t\right)$

E. $h(t) = 45 - 35\cos(8t)$

42. Find f^{-1} for the function $f(x) = 5 \cdot 2^{x-3} + 4.$

A. $f^{-1}(x) = \log(x+4) + 3$

B. $f^{-1}(x) = \log_2\left(\frac{x-4}{5}\right) + 3$

C. $f^{-1}(x) = \log_2(x-4) + \log_2 5 + 3$

D. $f^{-1}(x) = \log\left(\frac{x}{4}\right) + 3$

E. None of the above

43. Find the number of sides in a polygon whose interior angles sum to 2160°.

A. 12

B. 14

C. 10

D. 13

E. 15

44. Solve the determinant equation $\begin{vmatrix} x & 1 & 2 \\ 1 & x & 3 \\ 0 & 1 & 2 \end{vmatrix} = -4x$.

A. $\left\{\frac{-3+\sqrt{41}}{4}, \frac{-3-\sqrt{41}}{4}\right\}$

B. $\left\{-\frac{3}{4}+23i, -\frac{3}{4}-23i\right\}$

C. $\left\{0, \frac{3}{2}\right\}$

D. $\left\{0, -\frac{1}{2}\right\}$

E. None of the above

45. For $\sin\theta = 2\cot\theta$, find the value(s) of $\cos\theta$.

A. $-1+\sqrt{2}$

B. $-1\pm\sqrt{2}$

C. $-1-\sqrt{2}$

D. -1

E. $-2+2\sqrt{2}$