1. Simplify the following rational expression completely.

$$\frac{x^2-95x+2016}{x^2-1024}$$

Solution:

$$\frac{x^2-95x+2016}{x^2-1024} = \frac{(x-32)(x-63)}{(x-32)(x+32)} = \frac{x-63}{x+32}$$

Answer: $\boldsymbol{\frac{x-63}{x+32}}$

2. A line segment is randomly selected from the set of sides and diagonals of an n-sided regular polygon. Find the probability that the line segment is a side of the polygon.

Solution:

The total number of sides and diagonals is $\binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)(n-2)!}{2!(n-2)!} = \frac{n(n-1)}{2}$

$$P(choosing\ a\ side) = \frac{sides}{sides+diagonals} = \frac{n}{\frac{n(n-1)}{2}} = \frac{2}{n-1}$$

Answer: $\boldsymbol{\frac{2}{n-1}}$

3. Two fair 6-sided die with sides numbered 1 though 6 are rolled. It is observed that doubles (same number appearing on both dice) has *not* been rolled. Find the probability that a sum of 7 was rolled given that doubles was not rolled

Solution:

Total combinations of rolling 2 die = 36

Number of die that are not doubles = 6

Number of combinations that are not doubles = 30

Number of combinations that add to 7 = 6

Probability of a sum of 7 given the die is not even is 5/30 = 1/5

Answer: $\frac{1}{5}$

4. While traveling from his house to his grandmother's house, George fell asleep when he was half of the distance to her house. When George awoke, he still had to travel half the distance that he had traveled while sleeping. For what part of the entire distance had he been asleep?

Solution:

First Part of trip: ½, Fell asleep: x, Remaining part of the trip: ½ x

$$\frac{1}{2} + x + \frac{1}{2}x = 1$$

$$\frac{3}{2}x = \frac{1}{2}$$

$$x = \frac{1}{3}$$

Answer: **1/3**

5. It is now 3:30 PM. In 5,970 minutes from now, Amie will start playing her piano solo. What time of day will Amie start her piano solo?

5,970 minutes = 99 hours and 30 minutes

99mod(24) = 3 Which is the same as 6:30pm

adding the 30 minutes = 7pm

ANSWER: **7 PM**

6. Since his animals were crowded in the rectangular pasture, Old MacDonald decided to double the width and triple the length of the pasture. How many times as large is the newer pasture than the old one?

Solution:

Area = LW

New Area = (2w)(3L) = 6LW

ANSWER: **6**

7. What is the smallest positive angle θ satisfying the equation
$\cos^2\theta - \sin^2 2\theta = 0$

Solution: $\cos^2\theta - 4\sin^2\theta \cos^2\theta = 0$

$\cos^2\theta(1-4\sin^2\theta) = 0$

$\cos^2\theta = 0$ or $(1-4\sin^2\theta) = 0$

$\theta = 90^o$ or $\theta = 30^o$

ANSWER: 30^o

8. When Alice entered the Forest of Forgetfulness, she forgot the day of the week. She met the Lion and the Unicorn resting under a tree. The Lion lies on Monday, Tuesdays and Wednesdays and tells the truth on the other days of the week. The Unicorn, on the other hand, lies on Thursday, Fridays and Saturdays but tells the truth on the other days of the week. They made the following statements:

Lion: "Yesterday was one of my lying days."
Unicorn: "Yesterday was one of my lying days."
From these two statements, Alice was able to deduce the day of the week. What day was it?

Solution: If the Lion is telling the truth, the day of the week must be Thursday. If he is lying, then the day must be Monday. So the day of the week must be Thursday or Monday. If the Unicorn is telling the truth, the day of the week must be Sunday. If he is lying, then it must be Thursday. So, the day of the week has to be Thursday.

ANSWER: **THURSDAY**

9. The sum of three numbers is 17. The first number is twice the second number. The third number is 5 more than the second number. What is the value of the largest of the three numbers?

Solution: Let x denote the second number

First number = 2x

Third number = x + 5

Sum: 2x + x + x + 5 = 17, x = 3

First number = 6, Second number = 3 and Third number = 8

ANSWER: **8**

10. Who is the person?

Ken Keeler Graduated summa cum Laude with a bachelor's degree in applied mathematics from Harvard University in 1983. In 1990, he received his PhD in applied math from Harvard University. The title of his doctoral thesis was Map Representations and Optimal Encoding for Image Segmentation

He wrote for the Simpson's before leaving to write for Futurama.

The Futurama Theorem is a real life mathematical theorem written purely for the use in "The Prisoner of Benda"

It is the first known theorem to be created for the sole purpose of entertainment in a TV show.

The theorem proves that regardless of how many mind switches between two bodies have been made, they can still al be restored to their original bodies using only two extra people, provided these two people have not had any mid switches prior (assuming two people cannot switch minds back with each other after their original switch.)

ANSWER: Ken Keeler

11. One thousand perfect cubical blocks of the same size are stacked on a table as a 10in. by 10in. by 10in. cube. How many of the original blocks are hidden from view?

Solution: In view:

Two faces: 10" x 10" = 200

Two faces: 10" x 8" = 160

One face: 8" x 8" = 64

Total in view = 424

Hidden from view = 1000 – 424 = 576

ANSWER: **576**

12. Define a function f(n) on the positive integers by

$$f(n) = \begin{cases} 3n+1 & if\ n\ is\ even \\ 2n & if\ n\ is\ odd \end{cases}$$

What is f(f(f(f(3))))?

Solution:

f(3) = 2(3) = 6

f(6) = 3(6) + 1 = 19

f(19) = 2(19) = 38

f(38) = 3(38) + 1 = 115

ANSWER: **115**

13. What fraction in lowest terms is equal to the repeating decimal $0.4\overline{325}$?

Solution:

If $x = 0.4\overline{325}$, then $1000x = 432.5\overline{325}$ and $999x = 432.1$.
Thus $9990x = 4321$ and $x = \frac{4321}{9990}$.

ANSWER: $\frac{4321}{9990}$

14. Suppose that $2a3$ is a three-digit number which, when added to 326, gives the three-digit result $5b9$. If $5b9$ is divisible by 9, find the sum of a and b.

Solution:

If 5b9 is divisible by 9 then 5+b+9 =9n; since b is a single digit, b = 4

2a3+326 = 5b9

2a3 + 326 = 549

2a3 = 223; therefore a = 2

Sum = 2 + 4 = 6

ANSWER: **6**

15. If $h(x) = \frac{x}{x-1}$, find the value of

$$h\left(h\left(h\left(h\left(h(3+\sqrt{3})\right)\right)\right)\right).$$

Solution:

$h(3+\sqrt{3}) = 3-\sqrt{3}$ and $h(3-\sqrt{3}) = 3+\sqrt{3}$. Thus, repeated application of this function yields these alternating values, ending with $3-\sqrt{3}$.

ANSWER: $\mathbf{3-\sqrt{3}}$

16. If 60% of 70% of 80% of x is equal to 1.008, what is 50% of x?

Solution:

0.60(0.70(0.80x) = 1.008

x = 3

0.5(x) = .5(3) =1.5

ANSWER: **1.5**

17. What is the probability that at least 2 out of 4 people were born on the same day of the week?

Solution:

P(nobody was born on the same day) = (7/7)(6/7)(5/7)(4/7) = 120/343

P(At least two are born on the same day) =

1 – P(nobody was born on the same day) = 1 - 120/343 = 223/343

ANSWER: **223/343**

18. A manufacturer has three machines producing light bulbs. Machine A produces 40% of the light bulbs with 1% of them defective. Machine B produces 35% of the light bulbs with 2% being defective. Machine C produces 25% of the light bulbs with 4% being defective. If a randomly selected light bulb is tested and found defective, what is the probability that it was produced by Machine A? Round answer to three decimal places.

Solution:

Machine A (0.40(.01)) = 0.004 bulbs are defective

Machine B (0.35)(.02) = 0.007 Bulbs are defective

Machine C (0.25)(0.04) = .01 Bulbs are defective

Total number of defective = .004 + .007 + .01 = 0.021

Probability of selecting a light bulb from Machine A given the bulb is defective is

0.004 / 0.021 =0.190476

ANSWER: **.190**

19. Given that $f(2) = 5, f'(2) = -2, g(2) = -4$
$g'(2) = 5 \text{ and } h(x) = xf(x)g(x), \text{find } h'(2)$

Solution: $h(x) = xf(x)g(x)$

$$h'(x) = f(x)g(x) + xf'(x) + xf(x)g'(x)$$
$$h'(2) = f(2)g(2) + 2f'(2)g(2) + 2f(2)g'(2)$$
$$h'(2) = (5)(-4) + 2(-2)(-4) + 2(5)(5)$$
$$h'(2) = 46$$

ANSWER: ***46***

20. Who is this person?

Solution: There is no 16-Clue Sudoku: Solving the Sudoku Minimum Number of Clues Problem

Principal Investigator: Gary McGuire, Project Collaborator: Bastian Tugemann, Project Contributor: Gilles Civario, January 1, 2012

Abstract : We apply our new hitting set enumeration algorithm to solve the sudoku minimum number of clues problem, which is the following question: What is the smallest

number of clues (givens) that a sudoku puzzle may have? It was conjectured that the answer is 17. We have performed an exhaustive search for a 16-clue sudoku puzzle,

and we did not find one, thereby proving that the answer is indeed 17. This article describes our method and the actual search.

The hitting set problem is computationally hard; it is one of Karp's twenty-one

classic NP-complete problems. We have designed a new algorithm that allows us to efficiently enumerate hitting sets of a suitable size. Hitting set problems have

applications in many areas of science, such as bioinformatics and software testing.

School of Mathematical Sciences, University College Dublin, Ireland.

ANSWER: **Dr Gary McGuire**