1. The tree height, H(t), for a certain species of tree after "t" years is modeled by

$$H(t) = \frac{50}{1 + 45e^{-0.2t}}$$

How long will it take for the tree to reach a height of 30 ft?

A. $\ln\left(\frac{2}{135}\right)$	B. $-5\ln\left(\frac{2}{135}\right)$	C. $-.2\ln\left(\frac{2}{135}\right)$	D. $-\ln\left(\frac{2}{135}\right)$	E. None of the above

$(1 + 45e^{-.2t})30 = \frac{50}{1+45e^{-.2t}}(1 + 45e^{-.2t})$

$30 + 1350e^{-.2t} = 50$

$\ln e^{-.2t} = \ln 20/1350$

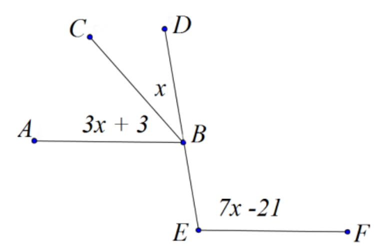
$t = \frac{ln\left(\frac{2}{135}\right)}{-.2} = -5ln\left(\frac{2}{135}\right)$ Correct Answer: B

2. The measures of the angles are as marked in the diagram. If $\overline{AB}$ is parallel to $\overline{EF}$, find the value of x.

A. 8°	B. 18°	C. 57°	D. 33°	E. 123°

Extending AB and EF, we see that DE is a transversal to the two parallel lines. We have, $\angle DBA + \angle DEF = 180°$. Substituting the values we have

$(3x+3+x)+(7x-21) = 180°, 11x-18 = 180°, 11x = 198°, x = 18°$



Correct Answer: B

3. Find the sum of four consecutive negative integers such that the sum of the squares of the first and fourth integer is 117

A. -14	B. -20	C. -30	D. -38	E. -42

Let x, x+1, x+2 and x+3 be the four consecutive negative integers
$x^2 + (x+3)^2 = 117$
$2x^2 + 6x - 108 = 0$
x = -9 or 6
Sum = -9 – 8 – 7 – 6 = -30 Correct Answer: C

4. At what point does the line normal to the curve $x^2y^3 + y + 2 = 0$ at (1,-1) intersect the line $2x - 3y + 7 = 0$?

A. (-1/2, 2)	B. (5, -1)	C. (4, 5)	D. (1/4, 1/2)	E. (17, -9)

$dy/dx = \dfrac{-2xy^3}{3x^2y^2+1}$ at (1,-1) $dy/dx = 2/4 = ½$, slope of normal = -2

$y = -2x + 1$

$2x - 3y = -7 \rightarrow 2x - 3y = -7$

$2x + y = 1 \quad\quad -2x - y = -1$

$-4y = -8$

$y = 2$

$2x - 3(2) = -7 \rightarrow 2x = -1 \rightarrow x = -½$

Correct Answer: A

5. Evaluate, $\cos^2\left(\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)\right)$.

A. 4/5	B. 9/10	C. 3/5	D. 7/10	E. 1/2

$$\cos^2\left(\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)\right) = \frac{1+\cos\left(\sin^{-1}\left(\frac{3}{5}\right)\right)}{2} = \frac{1+\frac{4}{5}}{2} = \frac{5+4}{10} = \frac{9}{10}$$

Correct Answer: B

6. A ski lift brings a skier to the summit in 15 minutes. In 4 ¼ hours an instructor makes 3 runs with the class and 2 runs alone. The instructor alone takes ¼ the time used when skiing with the class. How long does it take the instructor alone to ski a run?

A. 3/7 hour	B. 3/14 hour	C. 6/7 hour	D. 4/11 hour	E. 7/6 hour

x = time to ski down the hill with the class

5(¼) + 3x +2(¼ x) = 4 ¼

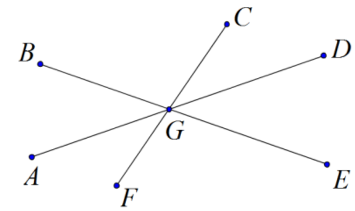
5/4 +7/2 x = 17/4, 7/2 x = 3, x = 6/7

Time down the hill alone = ¼ (6/7) = 3/14 hrs

Correct Answer: B

7. Three straight lines intersect at G and $\angle CGD = \angle DGE$ in the figure given. The ratio of the angle measure of $\angle CGB$ to $\angle BGF$ is 11:4. What is the angle measure of $\angle AGE$?

A. $12°$	B. $24°$	C. $48°$	D. $132°$	E. $156°$



$\angle CGB$ and $\angle BGF$ lie on a straight line, $\angle CGB + \angle BGF = 180°$. Using the ratio, we have $4x + 11x = 180°, 15x = 180°, x = 12°$. From this, $\angle BGF = 48°$ and $\angle CGB = 132°$. $\angle BGF$ and $\angle CGE$ are vertical angles. Since $\angle CGD = \angle DGE$, we have $\angle CGD = \angle DGE = 24°$. Since $\angle CGD$ and $\angle AGF$ are vertical angles and $\angle BGC$ and $\angle FGE$ are vertical angles we have $\angle AGE = \angle AGF + \angle FGE = 24° + 132° = 156°$

Correct Answer: E

8. If $r^2 + s^2 = 13, rs = 6$ and r,s < 0 , then find $\frac{1}{r} + \frac{1}{s}$

A. -7/12	B. -7/10	C. -2/3	D. -5/6	E. -1/2

Substitute r = 6/s, $\left(\frac{6}{s}\right)^2 + s^2 = 13$

$s^4 - 13s^2 + 36 = 0$, s = ±2, ±3

$\frac{1}{-2} + \frac{1}{-3} = -\frac{5}{6}$

Correct Answer: D

9. A mouse is at the bottom of a 10-foot-tall clock. The mouse climbs up at a constant rate of 3 feet per hour. But when the clock strikes at the hour, he falls back 1 foot. If the mouse starts climbing at 8am, at what time to the nearest minute will it reach the top of the clock?

A. 12:40 PM	B. 12:30 PM	C. 1:10 PM	D. 1:00 PM	E. 11:40 AM

First hour up 3 down one = 2 feet 9AM
Second hour 2+3-1 = 4 10AM
Third hour 4+3-1 = 6 11AM
Fourth hour 6 + 3 – 1 = 8 noon
Fifth hour 8 + 3 = 11; at the 5th hour the mouse only need to climb 2 feet. Since he climbs 3 feet in 60 minutes then 2 feet = 40 minutes.

Correct Answer: A

10. Find the smallest positive value of θ in the equation $\sin(\theta+1)=\cos\theta$.

A. $\frac{3\pi}{4}-\frac{1}{2}$	B. $\frac{\pi}{4}-\frac{\sqrt{3}}{2}$	C. $\frac{\pi}{2}-\frac{1}{2}$	D. $\frac{\pi}{4}-\frac{1}{2}$	E. $\frac{3\pi}{4}-\frac{\sqrt{3}}{2}$

By the co-function identity. $\cos\theta=\sin\left(\frac{\pi}{2}-\theta\right)$. Replacing the right hand side, we have $\sin(\theta+1)=\sin\left(\frac{\pi}{2}-\theta\right)$. Equating the arguments, we have $\theta+1=\frac{\pi}{2}-\theta, 2\theta=\frac{\pi}{2}-1, \theta=\frac{\pi}{4}-\frac{1}{2}$

Correct Answer: D

11. If $f(x)=ax^3+bx^2+cx+d$ and 2a = 3b, c = -9a and d = -9b, then find the sum of the zeros

A. -2/3	B. 14/3	C. 8	D. 0	E. 4

$0=ax^3+bx^2-9ax-9b$
$0=x^2(ax+b)-9(ax+b)$
$0=(x^2-9)(ax+b)$
x = -3, 3 and –b/a
Since 2a = 3b, b/a = 2/3

Correct Answer: A

12. Triangle ABC has vertex A at the origin, vertex B at the point (3,0), and vertex C on the circle with center (6,4) and radius 2. What is the maximum possible area for triangle ABC?

A. 3	B. 16	C. 18	D. 6	E. 9

The length between vertices A and B is 3, which is the base to the triangle. The height of the triangle is found by finding the highest point of the circle, which is the point (6,6). The y-coordinate acts as the height of the triangle, which is 6. Therefore, the area of triangle ABC is $\frac{1}{2}(3)(6)=9$

Correct Answer: E

13. A wholesale dealer figures that 20% of the receipts from the selling prices goes to overhead, 10% goes to commissions and 10% to profit. What is the markup on an item costing the wholesaler $120

A. $48	B. $200	C. $80	D. $100	E. $144

x = retail price of the item
120 + (.2 + .1 + .1) x = x
120 = .6x, x = 200
Markup = 200 – 120 = $80

Correct Answer: C

14. **(Tie Break No.1)** What is the number of pairs of positive integers (x, y) that satisfy $2x + 3y = 515$?

A. 46	B. 68	C. 86	D. 112	E. 52

If $x = 1$, then $y = 171$ (2(1) + 3(171) = 515). Also, if y = 1, then x = 256 (2(256) + 3(1) = 515)

y = 1, 3, 5, ………171, 171 = 1 + 2(n – 1)

n = 86

Correct Answer: C

15. Convert the rectangular equation into polar coordinates: $x^3 + xy^2 = y^2$.

A. $r = \sin\theta \tan\theta$	B. $r = \cos\theta \cot\theta$	C. $r = \sin\theta \cos\theta$	D. $r = \tan\theta$	E. $r = \csc\theta - 2$

$$x^3 + xy^2 = y^2, x\left(x^2 + y^2\right) = y^2, \left(r\cos\theta\right)r^2 = r^2\sin^2\theta, r^3\cos\theta - r^2\sin^2\theta = 0,$$

$$r^2\left(r\cos\theta - \sin^2\theta\right) = 0, r = 0 \text{ or } r\cos\theta - \sin^2\theta = 0, r\cos\theta = \sin^2\theta, r = \frac{\sin^2\theta}{\cos\theta},$$

$$r = \frac{\sin\theta}{\cos\theta}\sin\theta, r = \sin\theta\tan\theta.$$

Correct Answer: A

16. The linear term of a quadratic equation was incorrectly copied by a student who made no other mistake. The student found the roots of that equation to be 6 and -2. Another student made an error only in copying the constant term and found -5 and – 3 as the roots. What was the sum of the roots?

A. -8	B. -4	C. -16	D. 0	E. -12

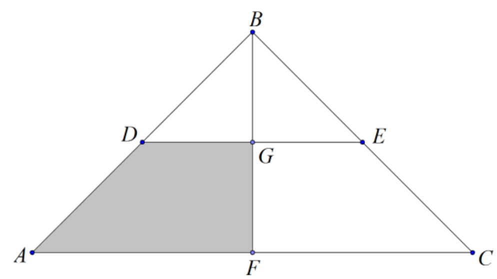
First student
(x – 6)(x + 2) = 0, $x^2 - 4x - 12 = 0$
Second Student
(x+5)(x+3) = 0, $x^2 + 8x + 15 = 0$
Actual
$x^2 + 8x - 12 = 0,\ x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-12)}}{2(1)}, x = -4 \pm 2\sqrt{7}.$

Correct Answer: A

17. The area of the triangle ABC in the figure is 16 square inches. Points D and E are midpoints of the congruent segments $\overline{AB}$ and $\overline{BC}$ respectively. Altitude $\overline{BF}$ bisects $\overline{AC}$. What is the area of the shaded region?

A. 3 sq. in	B. 4 sq. in	C. 6 sq. in	D. 8 sq. in	E. 12 sq. in

Since BF bisects AC, ΔABF and Δ FBC have equal area. Their areas are 8 square inches. Let G be the intersection of BF and DE. Since D is the midpoint of AB; DB = AB/2. Likewise, BE = BC/2, ΔBDE is similar to ΔABC. Then ΔBDE and ΔBAC are similar making DE parallel to AC. This makes ΔDBG similar to ΔABF. The ratio to the sides is ½. The ratio of their areas is ¼. So triangle BDG is ¼ the area of triangle BAF. Subtracting the areas we have 8 – 2 = 6.



Correct Answer: C

18. Air resistance causes the path of each swing (after the first) of a pendulum bob to be .9 as long as the preceding swing. If the path of the first swing is 20 inches long, what is the total distance traveled by the bob in coming to rest?

A. 150 inches	B. 200 inches	C. 250 inches	D. 300 inches	E. 350 inches

$S_\infty = \frac{a_1}{1-r}$
$S_\infty = \frac{20}{1-.9} = 200$

Correct Answer: B

19. Two numbers are chosen at random from the whole numbers from 1 to 20 without replacement. Find the probability that the two numbers are twin primes (primes that differ by 2).

A. 1/95	B. 2/95	C. 3/95	D. 4/95	E. 7/95

Of the (20)(19) = 380 possible outcomes for the two numbers, the following involve twin primes: (3,5), (5,7), (11,13), (17,19), (5,3), (7,5), (13,11), and (19,17).

Correct Answer: B

20. Given that $\cos 2x = \frac{2\sqrt{2}}{3}$, find the value of $\sin^4 x + \cos^4 x$.

A. 1/3	B. 13/18	C. 17/18	D. 5/6	E. 2/3

By the Pythagorean Identity, $\sin 2x = \sqrt{1-(\cos 2x)^2} = \sqrt{1-\left(\frac{2\sqrt{2}}{3}\right)^2} = \sqrt{1-\frac{8}{9}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$.

$$\cos 2x = \frac{2\sqrt{3}}{3}, \cos^2 x - \sin^2 x = \frac{2\sqrt{2}}{3}, (\cos^2 x - \sin^2 x)^2 = \left(\frac{2\sqrt{2}}{3}\right)^2, \cos^4 x + 2\sin^2 x\cos^2 x + \sin^4 x = \frac{8}{9},$$

$$\cos^4 x + \sin^4 x = \frac{8}{9} - 2\sin^2 x\cos^2 x, \cos^4 x + \sin^4 x = \frac{8}{9} + \frac{1}{2}\sin^2 2x, \cos^4 x + \sin^4 x = \frac{8}{9} + \frac{1}{2}\left(\frac{1}{3}\right)^2 = \frac{8}{9} + \frac{1}{18} = \frac{17}{18}$$

Correct Answer: C

21. Solve. $\begin{aligned} \ln x &= 4\ln y \\ \log_3 x &= 2 + 2\log_3 y \end{aligned}$

A. {(0, 0), (81, – 3), (81, 3)}	B. {(81, – 3)}	C. {(81, 3)}	D. { }	E. {(0, 0)}

$$\ln x = \ln y^4, x = y^4, \log_3 x - \log_3 y^2 = 2, \log_3 \frac{x}{y^2} = 2,$$

$$3^2 = \frac{x}{y^2}, 9y^2 = x, 9y^2 = y^4, 0 = y^4 - 9y^2, 0 = y^2(y+3)(y-3)$$

$$y = 3 \textit{ checks} \Rightarrow \{(81,3)\}$$

Correct Answer: C

22. **(Tie Break No. 3)** A triangle has side measures of 16 cm, 17 cm, and 17 cm. A second triangle is drawn with sides measuring 17 cm, 17cm, and x cm, where x is a whole number other than 16. If the two triangles have equal areas, what is the value of x?

A. 8 cm	B. 15 cm	C. 23 cm	D. 30 cm	E. 40 cm

The first triangle is isosceles. Dropping an altitude between the two 17 cm sides, we find that the altitude is 15 cm. Then, $A=\frac{1}{2}bh=\frac{1}{2}(16)(15)=120$. For the second triangle, drop an altitude and let that be the height. A right triangle can be formed with a legs, b/2 and h, and hypotenuse 17. We have $\left(\frac{b}{2}\right)^2+h^2=17^2$. The area of the second triangle is $120=\frac{1}{2}bh$ or $240=bh$. Plugging second equation into the first we have.

$$\frac{b^2}{4}+\frac{57600}{b^2}=289, b^4-1156b^2+230400=0, \left(b^2-900\right)\left(b^2-256\right)=0,$$

$$(b-30)(b+30)(b-16)(b+16)=0, b=-30,-16,16,30$$

The only valid solution is 30.

Correct Answer: D

23. An outdoor amphitheater has 35 seats in the first row, 37 in the second row, 39 in the third row, and so on. There are 27 rows altogether. How many can the amphitheater seat?

A. 1612 seats	B. 1560 seats	C. 87 seats	D. 1647 seats	E. None of the Above

$$a_1=35,\ d=2,\ n=27$$

$$S_n=\frac{n}{2}[2a_1+(n-1)d]$$

$$S_{27}=1647$$

Correct Answer: D

24. Suppose that the angle between the minute hand and hour hand of a clock is 60^0. If the minute hand is 16 inches long and the hour hand is 10 inches long, then what is the distance between the tip ends of the hands in inches?

A. 10 inches	B. 11 inches	C. 12 inches	D. 13 inches	E. 14 inches

Solution: Law of Cosines $d^2 = 10^2 + 16^2 - 2(10*16)\cos 60 = 196$; therefore, d =14

Correct Answer: E

25. Solve for *x*, $\frac{\pi}{2}+\sin^{-1}x=\frac{\sqrt{3}}{2}$

A. $\sin\frac{1}{2}$	B. $\frac{\pi}{3}$	C. $\frac{\pi}{6}$	D. $\frac{2\pi}{3}$	E. $-\cos\frac{\sqrt{3}}{2}$

$$\sin^{-1}x+\frac{\pi}{2}=\frac{\sqrt{3}}{2}, \sin^{-1}x=\frac{\sqrt{3}}{2}-\frac{\pi}{2}, x=\sin\left(\frac{\sqrt{3}}{2}-\frac{\pi}{2}\right),$$

$$x=\sin\frac{\sqrt{3}}{2}\cos\frac{\pi}{2}-\cos\frac{\sqrt{3}}{2}\sin\frac{\pi}{2}, x=\sin\frac{\sqrt{3}}{2}(0)-\cos\frac{\sqrt{3}}{2}(1)=-\cos\frac{\sqrt{3}}{2}$$

Correct Answer: E

26. Solve $\frac{3x-5}{x+2}\le 2$.

A. $(-2, 9)$	B. $[-2, 9]$	C. $(-2, 9]$	D. $(-\infty,-2)\cup(9,\infty)$	E. $(-\infty, \infty)$

$$\frac{3x-5}{x+2}-2\le 0$$

$$\frac{3x-5-2(x+2)}{x+2}\le 0$$

$$\frac{x-9}{x+2}\le 0$$

$$\frac{x-9}{x+2}=0$$

$x=9$ Correct Answer: C

$x+2=0$

$x=-2$

=> $(-\infty,-2)\,(-2,9)\,(9,\infty)$

$+ \quad - \quad +$

27. Find the area of the region enclosed by $|y|+|2x|=6$.

A. 36	B. 12	C. 6	D. 24	E. 18

$|y|+|2x|=6 \Rightarrow y=\begin{cases}6-|2x|\\|2x|-6\end{cases}$. So we have a rhombus with diagonals of length 12 and 6.

The area is $2\left(\frac{1}{2}bh\right)=2\left(\frac{1}{2}d_1\left(\frac{1}{2}d_2\right)\right)=\frac{d_1d_2}{2}=\frac{12\cdot 6}{2}=36$

Correct Answer: A

28. Solve $25^x - 8 \cdot 5^x = -16$.

A. $\{\log_5 4\}$	B. $\{\log_4 5\}$	C. {4}	D. {625}	E. { }

$$\left(5^x\right)^2 - 8 \cdot 5 + 16 = 0$$
$$let\ u = 5^x$$
$$u^2 - 8u + 16 = 0$$
$$(u-4)^2 = 0$$
$$u = 4$$
$$------------$$
$$4 = 5^x$$
$$\log_5 4 = x$$

Correct Answer: A

29. You are riding a Ferris wheel. Your height *h* (in feet) above the ground at any time *t* (in seconds) can be modeled by $h = 25\sin\frac{\pi}{15}(t-75)+30$. The Ferris wheel turns for 135 seconds before it stops to let the first passengers off. What are the minimum and maximum heights above the ground?

A. -25 ft., 25 ft.	B. 0ft., 25 ft.	C. 5 ft., 75 ft.	D. 5ft., 30 ft.	E. 5 ft., 55ft.

The amplitude of the function is 25 with a vertical shift up of 30. Minimum = –25 + 30 = 5. Maximum = 25 + 30 = 55.

Correct Answer: E

30. **(Tie Break No. 2)** A circle of radius 4 is centered at the origin; every second, its radius increases by 3 units. A second circle, of radius 12, is centered at (30,0); every second, its radius decreases by 1 unit. This process continues until the circles meet. At that time, the point (27,4) lies in which location?

A. on the first circle	B. on the second circle	C. inside the second circle	D. inside the first circle	E. between the circles

The circles begin 14 units apart on the *x*-axis. $x^2 + y^2 = 16, (x-30)^2 + y^2 = 144$. During every second, they come 2 units closer to each other (first increases by 3 and second decreases by 1). So, in 7 seconds they will meet (or touch on the *x*-axis). The first circle will have a radius of 4 + (7)(3) = 25 and the second circle will have a radius of 12 – 7 = 5. The new equations are $x^2 + y^2 = 625, (x-30)^2 + y^2 = 25$. The point (27,4) satisfies the second equation so it is on the second circle.

Correct Answer: B

31. Find $f^{-1}(x)$ for the function $f(x)=\dfrac{x^2+3}{3x^2}$, $x>0$

A.	B.	C.	D.	E..
$f^{-1}(x)=\pm\sqrt{\dfrac{3}{3x-1}}$	$f^{-1}(x)=\sqrt{\dfrac{3}{3x-1}}$	$f^{-1}(x)=-\sqrt{\dfrac{3}{3x-1}}$	$f^{-1}(x)=\dfrac{1}{3}+\dfrac{1}{x^2}$	$f^{-1}(x)=\dfrac{3}{3x-1}$

$$y=\frac{x^2+3}{3x^2}, x=\frac{y^2+3}{3y^2}, 3xy^2=y^2+3, 3xy^2-y^2=3,$$

$$y^2\left(3x-1\right)=3, y^2=\frac{3}{3x-1}, y=\sqrt{\frac{3}{3x-1}}, y>0$$

Correct Answer: B

32. The weight of an object on Earth varies inversely as the square of its distance from the center of the Earth. If an object weighs 300 pounds on the surface of the Earth (4000 miles from the center), what is the weight of the object if it is 800 miles above the Earth? Round to the nearest whole number

A. 208 pounds	B. 250 pounds	C. 1 pound	D. 392 pounds	E. None of the above

$$w=\frac{k}{d^2}$$

$$300=\frac{k}{4000^2}$$

$$k=4{,}800{,}000{,}000$$

$$-----------$$

$$w=\frac{4800000000}{4800^2}=208$$

Correct Answer: A

33. **(Tie Break No.4)** Find the sum of all real solutions of the given equation: $(x^2-6x+9)^{x^2-4x+3}=1$

A. 3	B. 1	C. 5	D. 7	E. 6

Case 1: $If\ x^2-6x+9\neq 0,\ then\ x^2-4x+3=0;\ (x-3)(x-1)=0;\ x=3,1;\ but\ x\neq 3.\ \therefore x=1$

Case 2: $x^2-6x+9=1;\ (x-4)(x-2)=0;\ x=4,2$

Therefore, there are three real solutions: 1, 2, and 4.

Therefore, the sum of all real solutions is 7.

Correct Answer: D

34. Write $\cos(\arcsin x + \arccos y)$ as an algebraic expression containing x and y.

A.	B.	C.	D.	E
$xy+\sqrt{1-x^2}\sqrt{1-y^2}$	$y\sqrt{1-x^2}+x\sqrt{1-y^2}$	$x\sqrt{1-x^2}+y\sqrt{1-y^2}$	$y\sqrt{1-x^2}-x\sqrt{1-y^2}$	$x\sqrt{1-x^2}-y\sqrt{1-y^2}$.

$\cos(\arcsin x + \arccos y) = \cos(A+B) = \cos A \cos B - \sin A \sin B$ where

$A = \arcsin x, \sin A = x, \cos A = \sqrt{1-x^2}, B = \arccos y, \cos B = y, \sin B = \sqrt{1-y^2}$ so

$\cos(\arcsin x + \arccos y) = \cos(A+B) = \cos A \cos B - \sin A \sin B = y\sqrt{1-x^2} - x\sqrt{1-y^2}$.

Correct Answer: D

35. Solve the determinant equation $\begin{vmatrix} x & x+1 & x+2 \\ 2 & 3 & -1 \\ 3 & -2 & 4 \end{vmatrix} = 0$

A. { }	B. -13/6	C. 37/14	D. -37/14	E. -21/2

$$3\begin{vmatrix} x+1 & x+2 \\ 3 & -1 \end{vmatrix} - (-2)\begin{vmatrix} x & x+2 \\ 2 & -1 \end{vmatrix} + 4\begin{vmatrix} x & x+1 \\ 2 & 3 \end{vmatrix} = 0$$

$$3[-(x+1)-3(x+2)]+2[-x-2(x+2)]+4[3x-2(x+1)]=0$$

$$x = -\frac{37}{14}$$

Correct Answer: D

36. Simplify the following: $(\tan A + \tan B)(1 - \cot A \cot B) + (\cot A + \cot B)(1 - \tan A \tan B)$.

A. -1	B. $A+B$	C. 0	D. $A-B$	E. 1

$$(\tan A + \tan B)(1 - \cot A \cot B) + (\cot A + \cot B)(1 - \tan A \tan B)$$

$$\left(\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}\right)\left(1 - \frac{\cos A \cos B}{\sin A \sin B}\right) + \left(\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}\right)\left(1 - \frac{\sin A \sin B}{\cos A \cos B}\right)$$

$$\left(\frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B} - \frac{\sin A \cos B + \sin B \cos A}{\sin A \sin B}\right) + \left(\frac{\cos A \sin B + \cos B \sin A}{\sin A \sin B} - \frac{\cos A \sin B + \cos B \sin A}{\cos A \cos B}\right) = 0$$

Correct Answer: C

37. Three congruent rectangles are placed to form a larger rectangle as shown, with an area of 1350 cm^2. Find the area of a square that has the same perimeter as that of the larger rectangle (formed by the three congruent rectangles).

A. 900.00 cm^2	B. 450.00 cm^2	C. 1460.25 cm^2	D. 225.50 cm^2	E. 2025.75 cm^2

If the area of the larger rectangle is 1350 cm^2 then the area of each smaller congruent rectangle is 450 cm^2. For each small congruent rectangle the $l = 2w$. So the area of each smaller rectangle is $450 = lw = 2w^2, w = 15, l = 30.$ Thus the perimeter of the large rectangle is $3l + 4w = 150$. So the perimeter of the square is 150 cm making each side 37.5 cm and the area of the square is then $(37.5)^2 =$ 1406.25 cm^2.

Correct Answer: C

38. Write $\left[2\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)\right]^3$ into standard form, $a + bi$.

A. $4 + 4\sqrt{3}\ i$	B. $8\sqrt{3} + 8i$	C. $8 + 8\sqrt{3}\ i$	D. $8 + 8i$	E. $4\sqrt{3} + 4i$

$$\left[2\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)\right]^3 = 2^3\left[\cos\left(3\cdot\frac{\pi}{9}\right) + i\sin\left(3\cdot\frac{\pi}{9}\right)\right] = 8\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right] = 8\left[\frac{1}{2} + i\frac{\sqrt{3}}{2}\right] = 4 + 4i\sqrt{3}$$

Correct Answer: A

39. Let $f(x)$ be a function such that $f(1) = 1$ and $f(n) = n + f(n-1)$ for all natural numbers $n \geq 2$, find the value of n such that $f(4n) = 12f(n)$.

A. 8	B. 6	C. 4	D. 2	E. 0

$f(n) = n + f(n-1) = n + (n-1) + f(n-2) = n + (n-1) + (n-2) + f(n-3) = \cdots = n + (n-1) + (n-2) + \cdots + \big(n - (n-2)\big) + f(1) = n(n-1) - \big(1 + 2 + 3 + \cdots + (n-2)\big) + f(1) = \frac{n(n+1)}{2}\ \therefore f(n) = \frac{n(n+1)}{2}$

$f(4n) = 12f(n);\ \dfrac{4n(4n+1)}{2} = 12 * \dfrac{n(n+1)}{2};\quad 16n^2 + 4n = 12n^2 + 12n;$

$4n(n-2) = 0;\ \ n = 0\ or\ 2, but\ n \geq 2, therefore\ n = 2$

Correct Answer: D

40. The total number of interior angles in two regular polygons is 17 and the total number of diagonals is 53. How many sides does each regular polygon have?

A. 12 and 5	B. 10 and 7	C. 9 and 8	D. 13 and 4	E. 11 and 6

Let a and b represent the numbers of sides of the two regular polygons, then $a + b = 17$ and $\frac{a(a-3)}{2} + \frac{b(b-3)}{2} = 53.$ Solving by substitution, $a = 11$ and $b = 6$ (or vice versa).

Correct Answer: E

41. In a certain examination it is noted that the average score of those passing is 65 while the average score of those failing is 35. If the average of all participants is 53, what percentage of the participants passed?

A. 40%	B. 65%	C. 35%	D. 50%	E. 60%

Let $p = \#students\ passing\ \ and\ f = \#students\ failing$

Total score of all students passing the exam = $65p$

Total score of all students failing = $35f$

Total score of all students = $53(p + f)$

$\therefore 65p + 35f = 53p + 53f;\ \ 12p = 18f; p = 1.5f$

$\therefore percent\ of\ students\ passing = \frac{1.5f}{1.5f + f} \times 100\% = 60\%$

Correct Answer: E

42. The Montauk Point Lighthouse on Long Island has dual beams (two light sources opposite each other). Ships at sea observe a blinking light every 5 seconds. What rotation speed is required to do this (in revolutions/second)?

A. $\frac{1}{5}$ rev/sec	B. $\frac{1}{10}$ rev/sec	C. $\frac{2}{5}$ rev/sec	D. $\frac{1}{20}$ rev/sec	E. $\frac{1}{12}$ rev/sec

$$\omega = \frac{\theta}{t} = \frac{\pi}{5}\frac{rad.}{\text{sec.}} \times \frac{1}{2\pi}\frac{rev.}{rad.} = \frac{1}{10}\frac{rev.}{\text{sec.}}$$

Correct Answer: B

43. **(Tie Break No.5)** If $\frac{(a-b)(c-d)}{(b-c)(d-a)} = -\frac{5}{3}$, find $\frac{(a-c)(b-d)}{(a-b)(c-d)}$

A. 5/8	B. 3/5	C. 8/5	D. -8/5	E. 2/5

$(a-c)(b-d) = ab - ad - bc + cd;\ \ (b-c)(d-a) = bd - ab - cd + ac$

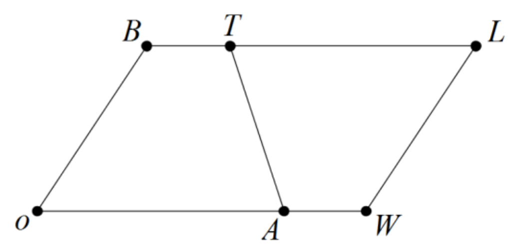
$\therefore (a-c)(b-d) = (ab+cd) - ad - bc$

$= bd + ac - (b-c)(d-a) - ad - bc$

$= (a-b)(c-d) - (b-c)(d-a)$

$\therefore \frac{(a-c)(b-d)}{(a-b)(c-d)} = 1 - \frac{(b-c)(d-a)}{(a-b)(c-d)} = 1 - \left(-\frac{3}{5}\right) = \frac{8}{5}$ Correct Answer: C

44. $BOWL$ is a parallelogram in which AT is 12. $BT = (1/3)BL$ and $AW = (1/3)OW$. If the perimeter of $BOAT$ is 40, find the perimeter of $BOWL$.



A. 66 units	B. 56 units	C. 28 units	D. 80 units	E. 60 units

BT + 12 + *OA* + *BO* = 40. *BT* + *OA* + *BO* = 28, *BT* = *AW*, *AW* + *OA* = *OW* so *OW* + *BO* = 28 meaning *BL* + *LW* = 28, so the perimeter of *BOWL* is 28 + 28 = 56.

Correct Answer: B

45. Find the simplest form for R, where $R = \sqrt{1+\sqrt{-3}} + \sqrt{1-\sqrt{-3}}$.

A. 2	B. $\sqrt{2}$	C. $2+\sqrt{-2}$	D. $2-\sqrt{-2}$	E. $\sqrt{6}$

Let $a = \sqrt{1+\sqrt{-3}}$ *and* $b = \sqrt{1-\sqrt{-3}}$.

Then, $R = a + b;\ \ R^2 = a^2 + b^2 + 2ab;$

$R^2 = \left(1+\sqrt{-3}\right) + \left(1-\sqrt{-3}\right) + 2\sqrt{\left(1+\sqrt{-3}\right)\left(1-\sqrt{-3}\right)}$

$R^2 = 2 + \sqrt{1+3} = 6;\ \ R = \sqrt{6}$

Correct Answer: E