

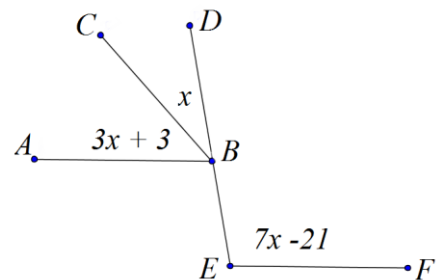
Math Bowl Written Test 2016

1. The tree height, $H(t)$, for a certain species of tree after “ t ” years is modeled by

$$H(t) = \frac{50}{1 + 45e^{-0.2t}}$$

How long will it take for the tree to reach a height of 30 ft?

- A. $\ln\left(\frac{2}{135}\right)$
 B. $-5\ln\left(\frac{2}{135}\right)$
 C. $-.2\ln\left(\frac{2}{135}\right)$
 D. $-\ln\left(\frac{2}{135}\right)$
 E. None of the above
2. The measures of the angles are as marked in the diagram. If \overline{AB} is parallel to \overline{EF} , find the value of x .



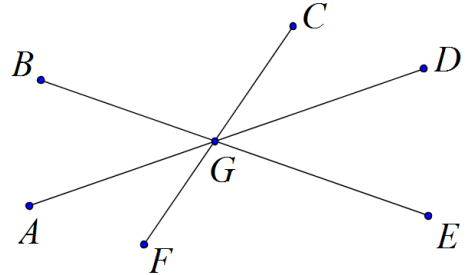
- A. 8°
 B. 18°
 C. 57°
 D. 33°
 E. 123°
3. Find the sum of four consecutive negative integers such that the sum of the squares of the first and fourth integer is 117.
- A. -14
 B. -20
 C. -30
 D. -38
 E. -42

Math Bowl Written Test 2016

4. At what point does the line normal to the curve $x^2y^3 + y + 2 = 0$ at $(1,-1)$ intersect the line $2x - 3y + 7 = 0$?
- A. $(-1/2, 2)$
 - B. $(5,-1)$
 - C. $(4,5)$
 - D. $(1/4, 1/2)$
 - E. $(17, -9)$
5. Evaluate $\cos^2\left(\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)\right)$.
- A. $4/5$
 - B. $9/10$
 - C. $3/5$
 - D. $7/10$
 - E. $1/2$
6. A ski lift brings a skier to the summit in 15 minutes. In $4\frac{1}{4}$ hours an instructor makes 3 runs with the class and 2 runs alone. The instructor alone takes $\frac{1}{4}$ the time used when skiing with the class. How long does it take the instructor alone to ski a run?
- A. $3/7$ hour
 - B. $3/14$ hour
 - C. $6/7$ hour
 - D. $4/11$ hour
 - E. $7/6$ hour

7. Three straight lines intersect at G and $\angle CGD = \angle DGE$ in the figure given. The ratio of the angle measure of $\angle CGB$ to $\angle BGF$ is $11:4$. What is the angle measure of $\angle AGE$?

- A. 12°
- B. 24°
- C. 48°
- D. 132°
- E. 156°



8. If $r^2 + s^2 = 13$, $rs = 6$ and $r, s < 0$, then find $\frac{1}{r} + \frac{1}{s}$

- A. $-7/12$
- B. $-7/10$
- C. $-2/3$
- D. $-5/6$
- E. $-1/2$

9. A mouse is at the bottom of a 10-foot-tall clock. The mouse climbs up at a constant rate of 3 feet per hour. But when the clock strikes at the hour, he falls back 1 foot. If the mouse starts climbing at 8am, at what time to the nearest minute will it reach the top of the clock?

- A. 12:40 PM
- B. 12:30 PM
- C. 1:10 PM
- D. 1:00 PM
- E. 11:40 AM

10. Find the smallest positive value of θ in the equation $\sin(\theta+1) = \cos \theta$.

- A. $\frac{3\pi}{4} - \frac{1}{2}$
- B. $\frac{\pi}{4} - \frac{\sqrt{3}}{2}$
- C. $\frac{\pi}{2} - \frac{1}{2}$
- D. $\frac{\pi}{4} - \frac{1}{2}$
- E. $\frac{3\pi}{4} - \frac{\sqrt{3}}{2}$

11. If $f(x) = ax^3 + bx^2 + cx + d$ and $2a = 3b$, $c = -9a$ and $d = -9b$, then find the sum of the zeros

- A. $-2/3$
- B. $14/3$
- C. 8
- D. 0
- E. 4

12. Triangle ABC has vertex A at the origin, vertex B at the point (3,0), and vertex C on the circle with center (6,4) and radius 2. What is the maximum possible area for triangle ABC?

- A. 3
- B. 16
- C. 18
- D. 6
- E. 9

13. A wholesale dealer figures that 20% of the receipts from the selling prices goes to overhead, 10% goes to commissions and 10% to profit. What is the markup on an item costing the wholesaler \$120?

- A. \$48
- B. \$200
- C. \$80
- D. \$100
- E. \$144

14. **(Tie Break No.1)** What is the number of pairs of positive integers (x, y) that satisfy $2x + 3y = 515$?

- A. 46
- B. 68
- C. 86
- D. 112
- E. 52

15. Convert the rectangular equation into polar coordinates: $x^3 + xy^2 = y^2$.

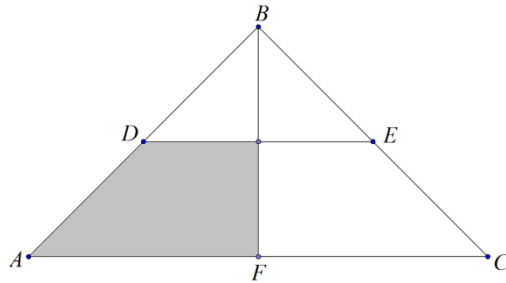
- A. $r = \sin \theta \tan \theta$
- B. $r = \cos \theta \cot \theta$
- C. $r = \sin \theta \cos \theta$
- D. $r = \tan \theta$
- E. $r = \csc \theta - 2$

16. The linear term of a quadratic equation was incorrectly copied by a student who made no other mistake. The student found the roots of that equation to be 6 and -2. Another student made an error only in copying the constant term and found -5 and -3 as the roots. What was the sum of the roots?

- A. -8
- B. -4
- C. -16
- D. 0
- E. -12

17. The area of the triangle ABC in the figure is 16 square inches. Points D and E are midpoints of the congruent segments \overline{AB} and \overline{BC} respectively. Altitude \overline{BF} bisects \overline{AC} . What is the area of the shaded region?

- A. 3 sq. in
- B. 4 sq. in
- C. 6 sq. in
- D. 8 sq. in
- E. 12 sq. in



18. Air resistance causes the path of each swing (after the first) of a pendulum bob to be .9 as long as the preceding swing. If the path of the first swing is 20 inches long, what is the total distance traveled by the bob in coming to rest?

- A. 150 inches
- B. 200 inches
- C. 250 inches
- D. 300 inches
- E. 350 inches

19. Two numbers are chosen at random from the whole numbers from 1 to 20 without replacement. Find the probability that the two numbers are twin primes (primes that differ by 2).

- A. $1/95$
- B. $2/95$
- C. $3/95$
- D. $4/95$
- E. $7/95$

20. Given that $\cos 2x = \frac{2\sqrt{2}}{3}$, find the value of $\sin^4 x + \cos^4 x$.

- A. $1/3$
- B. $13/18$
- C. $17/18$
- D. $5/6$
- E. $2/3$

21. Solve.
$$\begin{aligned} \ln x &= 4 \ln y \\ \log_3 x &= 2 + 2 \log_3 y \end{aligned}$$

- A. $\{(0, 0), (81, -3), (81, 3)\}$
- B. $\{(81, -3)\}$
- C. $\{(81, 3)\}$
- D. $\{ \}$
- E. $\{(0, 0)\}$

22. **(Tie Break No. 3)** A triangle has side measures of 16 cm, 17 cm, and 17 cm. A second triangle is drawn with sides measuring 17 cm, 17cm, and x cm, where x is a whole number other than 16. If the two triangles have equal areas, what is the value of x ?
- A. 8 cm
 - B. 15 cm
 - C. 23 cm
 - D. 30 cm
 - E. 40 cm
23. An outdoor amphitheater has 35 seats in the first row, 37 in the second row, 39 in the third row, and so on. There are 27 rows altogether. How many can the amphitheater seat?
- A. 1612 seats
 - B. 1560 seats
 - C. 87 seats
 - D. 1647 seats
 - E. None of the above
24. Suppose that the angle between the minute hand and hour hand of a clock is 60° . If the minute hand is 16 inches long and the hour hand is 10 inches long, then what is the distance between the tip ends of the hands in inches?
- A. 10 inches
 - B. 11 inches
 - C. 12 inches
 - D. 13 inches
 - E. 14 inches

25. Solve for x , $\frac{\pi}{2} + \sin^{-1} x = \frac{\sqrt{3}}{2}$
- A. $\sin \frac{1}{2}$
 - B. $\frac{\pi}{3}$
 - C. $\frac{\pi}{6}$
 - D. $\frac{2\pi}{3}$
 - E. $-\cos \frac{\sqrt{3}}{2}$
26. Solve $\frac{3x-5}{x+2} \leq 2$.
- A. $(-2, 9)$
 - B. $[-2, 9]$
 - C. $(-2, 9]$
 - D. $(-\infty, -2) \cup (9, \infty)$
 - E. $(-\infty, \infty)$
27. Find the area of the region enclosed by $|y| + |2x| = 6$.
- A. 36
 - B. 12
 - C. 6
 - D. 24
 - E. 18

28. Solve $25^x - 8 \cdot 5^x = -16$.
- A. $\{\log_5 4\}$
 - B. $\{\log_4 5\}$
 - C. $\{4\}$
 - D. $\{625\}$
 - E. $\{ \}$
29. You are riding a Ferris wheel. Your height h (in feet) above the ground at any time t (in seconds) can be modeled by $h = 30 + 25 \sin \left[\frac{\pi}{15}(t - 75) \right]$. The Ferris wheel turns for 135 seconds before it stops to let the first passengers off. What are the minimum and maximum heights above the ground?
- A. Minimum = -25 ft., Maximum = 25 ft.
 - B. Minimum = 0 ft., Maximum = 25 ft.
 - C. Minimum = 5 ft., Maximum = 75 ft.
 - D. Minimum = 5 ft., Maximum = 30 ft.
 - E. Minimum = 5 ft., Maximum = 55 ft.
30. **(Tie Break No. 2)** A circle of radius 4 is centered at the origin; every second, its radius increases by 3 units. A second circle, of radius 12, is centered at (30,0); every second, its radius decreases by 1 unit. This process continues until the circles meet. At that time, the point (27,4) lies in which location?
- A. on the first circle
 - B. on the second circle
 - C. inside the second circle
 - D. inside the first circle
 - E. between the circles

31. Find $f^{-1}(x)$ for the function $f(x) = \frac{x^2 + 3}{3x^2}$, $x > 0$

A. $f^{-1}(x) = \pm \sqrt{\frac{3}{3x-1}}$

B. $f^{-1}(x) = \sqrt{\frac{3}{3x-1}}$

C. $f^{-1}(x) = -\sqrt{\frac{3}{3x-1}}$

D. $f^{-1}(x) = \frac{1}{3} + \frac{1}{x^2}$

E. $f^{-1}(x) = \frac{3}{3x-1}$

32. The weight of an object on Earth varies inversely as the square of its distance from the center of the Earth. If an object weighs 300 pounds on the surface of the Earth (4000 miles from the center), what is the weight of the object if it is 800 miles above the Earth? Round to the nearest whole number

A. 208 pounds

B. 250 pounds

C. 1 pound

D. 392 pounds

E. None of the above

33. **(Tie Break No.4)** Find the sum of all real solutions of the given equation:

$$(x^2 - 6x + 9)^{x^2 - 4x + 3} = 1$$

A. 3

B. 1

C. 5

D. 7

E. 6

34. Write $\cos(\arcsin x + \arccos y)$ as an algebraic expression containing x and y .

A. $xy + \sqrt{1-x^2}\sqrt{1-y^2}$

B. $y\sqrt{1-x^2} + x\sqrt{1-y^2}$

C. $x\sqrt{1-x^2} + y\sqrt{1-y^2}$

D. $y\sqrt{1-x^2} - x\sqrt{1-y^2}$

E. $x\sqrt{1-x^2} - y\sqrt{1-y^2}$

35. Solve the determinant equation $\begin{vmatrix} x & x+1 & x+2 \\ 2 & 3 & -1 \\ 3 & -2 & 4 \end{vmatrix} = 0$

A. $\{ \}$

B. $-\frac{13}{6}$

C. $\frac{37}{14}$

D. $-\frac{37}{14}$

E. $-\frac{21}{2}$

36. Simplify the following: $(\tan A + \tan B)(1 - \cot A \cot B) + (\cot A + \cot B)(1 - \tan A \tan B)$.

A. -1

B. $A+B$

C. 0

D. $A-B$

E. 1

37. Three congruent rectangles are placed to form a larger rectangle as shown, with an area of 1350 cm^2 . Find the area of a square that has the same perimeter as that of the larger rectangle (formed by the three congruent rectangles).

- A. 900.00 cm^2
- B. 450.00 cm^2
- C. 1406.25 cm^2
- D. 225.50 cm^2
- E. 2025.75 cm^2



38. Write $\left[2\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)\right]^3$ into standard form, $a + bi$.

- A. $4 + 4\sqrt{3}i$
- B. $8\sqrt{3} + 8i$
- C. $8 + 8\sqrt{3}i$
- D. $8 + 8i$
- E. $4\sqrt{3} + 4i$

39. Let $f(x)$ be a function such that $f(1) = 1$ and $f(n) = n + f(n - 1)$ for all natural numbers $n \geq 2$, find the value of n such that $f(4n) = 12f(n)$.

- A. 8
- B. 6
- C. 4
- D. 2
- E. 0

Math Bowl Written Test 2016

40. The total number of interior angles in two regular polygons is 17 and the total number of diagonals is 53. How many sides does each regular polygon have?
- A. 12 and 5
 - B. 10 and 7
 - C. 9 and 8
 - D. 13 and 4
 - E. 11 and 6
41. In a certain examination it is noted that the average score of those passing is 65 while the average score of those failing is 35. If the average of all participants is 53, what percentage of the participants passed?
- A. 40%
 - B. 65%
 - C. 35%
 - D. 50%
 - E. 60%
42. The Montauk Point Lighthouse on Long Island has dual beams (two light sources opposite each other). Ships at sea observe a blinking light every 5 seconds. What rotation speed is required to do this (in revolutions/second)?
- A. $\frac{1}{5}$ revolutions/second
 - B. $\frac{1}{10}$ revolutions/second
 - C. $\frac{2}{5}$ revolutions/second
 - D. $\frac{1}{20}$ revolutions/second
 - E. $\frac{1}{12}$ revolutions/second

Math Bowl Written Test 2016

43. **(Tie Break No.5)** If $\frac{(a-b)(c-d)}{(b-c)(d-a)} = -\frac{5}{3}$, find $\frac{(a-c)(b-d)}{(a-b)(c-d)}$

A. $\frac{5}{8}$

B. $\frac{3}{5}$

C. $\frac{8}{5}$

D. $-\frac{8}{5}$

E. $\frac{2}{5}$

44. *BOWL* is a parallelogram in which *AT* is 12. $BT = (1/3)BL$ and $AW = (1/3)OW$. If the perimeter of *BOAT* is 40, find the perimeter of *BOWL*.

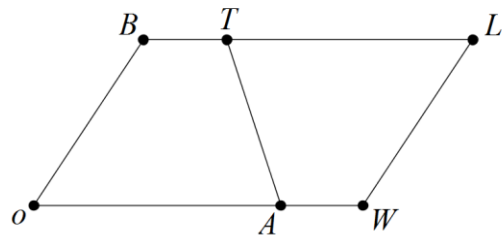
A. 66 units

B. 56 units

C. 28 units

D. 80 units

E. 60 units



45. Find the simplest form for *R*, where $R = \sqrt{1 + \sqrt{-3}} + \sqrt{1 - \sqrt{-3}}$.

A. 2

B. $\sqrt{2}$

C. $2 + \sqrt{-2}$

D. $2 - \sqrt{-2}$

E. $\sqrt{6}$